Celestial Navigation in a Teacup

Formerly "The Armchair Celestial Navigator"

Concepts, Math, History, the Works, but Different

Teacup Navigation

Rodger E. Farley

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Preface

Growing up I had always been fascinated by the thought of navigating by the stars. However, it instinctively seemed to me an art beyond my total understanding. Why, I don't know other than celestial navigation has always had a shroud of mystery surrounding it (no doubt to keep the hands from mutiny). Some time in my 40s I began to discard my preconceived notions regarding things that required 'natural' talent, and thus I began a journey of discovery. This book represents my efforts at teaching myself 'celestial', although it is not comprehensive of all my studies in this field. Like most educational endeavors one may sometimes plunge too deeply in seeking arcane knowledge and risk losing the interest and attention of the reader. With that in mind this book is dedicated simply to removing the cloak of mystery; to teach the concepts, some interesting history, the techniques, and computational methods using the simple pocket scientific calculator (or better yet make your own navigation software). And yes, also how to build your own navigational tools.

My intention is for this to be used as a self-teaching tool for those who have a desire to learn celestial from the intuitive, academic, and practical points of view. This book should also interest experienced navigators who are tired of simply 'turning the crank' with tables and would like a better behind-the-scenes knowledge. With the prevalence of hand electronic calculators, the traditional methods of using sight-reduction tables with pre-computed solutions will hardly be mentioned here. I am referring to the typical Hydrographic Office methods H.O. 249 and H.O. 229. Rather, the essential background and equations to the solutions will be presented such that the reader can calculate the answers precisely with a hand calculator and understand the *why*. You will need a scientific calculator, those having trigonometric functions and their inverse functions. Programmable graphing calculators such as the TI-86 and TI-89 are excellent for the methods described in the book. To those readers familiar with 'celestial', they will notice that I have departed the usual norms found in celestial navigation texts. I use a consistent sign convention which allows me to discard same-name and opposite-name rules.

Rodger Farley 2002

Variable and Acronym List

Hs	Altitude angle as reported on the sextant scale
Ha	Apparent altitude angle
Ho	Observed, or true altitude angle
Hc	Calculated altitude angle
IC	Index correction
SD	Angular semi-diameter of sun or moon
UL	Upper limb of sun or moon
LL	Lower limb of sun or moon
GHA	Greenwich hour angle
GHA_{hour}	Greenwich hour angle as tabulated at a specific integer hour
DEC	Declination angle
DEC_{hour}	Declination angle as tabulated at a specific integer hour
SHA	Sidereal hour angle
LHA	Local hour angle
Zo	Uncorrected azimuth angle
Zn	Azimuth angle from true north
v	Hourly variance from the nominal GHA rate, arcmin per hour
d	Hourly declination rate, arcmin per hour
h_{eye}	Eye height above the water, meters
$Corr^{DIP}$	Correction for dip of the horizon due to eye height
$Corr^{v}$	Correction to the tabular GHA for the variance v
$Corr^{d}$	Correction to the tabular declination using rate d
$Corr^{GHA}$	Correction to the tabular GHA for the minutes and seconds
$Corr^{ALT}$	Correction to the sextant altitude for atmospheric refraction, but parallax and semi-diameter are frequently added in
R	Correction for atmospheric refraction alone
D_{offset}	Offset distance using the intercept method, nautical miles
LAT	Latitude
LON	Longitude
LAT_{A}	Assumed latitude
LON_{A}	Assumed longitude
LAT_{DR}	Estimated latitude, or dead-reckoning latitude
LON_{DR}	Estimated longitude, or dead-reckoning longitude
LOP	Line of position
LAN	Local apparent noon
LMT	Local mean time

Chapter One   *Early Related History*

Why 360 degrees in a circle?

If you were an early astronomer you would have noticed that the stars rotate counterclockwise (ccw) about Polaris at the rate of seemingly once per day. And that as the year moved on the constellation's position would slowly crank around as well, once per year ccw. The planets were mysterious and thought to be gods as they roamed around the night sky, only going thru certain constellations named the zodiac (in the ecliptic plane). You would have noticed that after ¼ of a year had passed, or ~ 90 days, that the constellation had turned ccw about ¼ of a circle. It would have seemed that the angle of rotation per day was 1/90 of a quarter circle. A degree could be thought of as a heavenly angular unit, which is quite a coincidence with the Babylonian base 60 number system which established the angle of an equilateral triangle as 60º.

The Egyptians had divided the day into 24 hours, and the Mesopotamians further divided the hour into 60 minutes, 60 seconds per minute. It is easy to see the analogy between angle and clock time, since the angle was further divided into 60 arcminutes per degree, and 60 arcseconds per arcminute. An arcminute of a great circle on the surface of our planet defined the unit of distance; a nautical mile, which = 1.15 statute miles. By the way, mile comes from the Latin *milia* for 1000 double paces of a Roman soldier.

Size of the Earth

In the Near East during the 3rd century BC lived an astronomer-philosopher by the name of Eratosthenes, who was the director of the Egyptian Great Library of Alexandria. In one of the scroll books he read that on the summer solstice June 21 in Syene (south of Alexandria), one could see the sun's reflection at the bottom of deep wells (on tropic of Cancer). He wondered that on the same day in Alexandria, a stick would cast a measurable shadow. The ancient Greeks had hypothesized that the earth was round, and this observation by Eratosthenes confirmed the curvature of the Earth. But how big was it? On June 21 he measured the angle cast by the stick and saw that it was approximately 1/50th of a full circle (7.2 degrees). He hired a man to pace out the distance between Alexandria and Syene, who reported it was 500 miles. If 500 miles was the arc length for 1/50 of a huge circle, then the Earth's circumference would be 50 times longer, or 50 · 500 = 25000 miles. Simple tools and an enlightened mind can produce extraordinary results, considering he was less than 1% off. Amazingly a stick can also tell you the length of a year, 1st day of spring and fall (and summer and winter too), and the tilt of the earth's axis. Think about it. How would you do it?

Calendar

Very early calendars were based on the lunar month, 29 ½ days. This produced a 12-month year with only 354 days. Unfortunately, this would 'drift' the seasons backwards 11 ¼ days every year according to the old lunar calendars. Julius Caesar abolished the lunar year, used instead the position of the sun and fixed the true year at 365 ¼ days, and decreed a leap day every 4 years to make up for the ¼ day loss per Julian year of 365 days. Their astronomy was not accurate enough to know that a tropical year is 365.2424 days long; 11 minutes and 14 seconds shorter than 365 ¼ days. This difference adds a day every 128.2 years, so in 1582 the Gregorian calendar was instituted in which 10 days that particular October were dropped to resynchronize the calendar with the seasons, and 3 leap year days would not be counted every 400 years to maintain synchronicity.

Early Navigation

The easiest form of navigating was to never leave sight of the coast. Species of fish and birds, and the color and temperature of the water gave clues, as well as the composition of the bottom. When one neared the entrance to the Nile on the Mediterranean, the bottom became rich black, indicating that you should turn south. Why venture out into the deep blue water? Because of coastal pirates, and storms that pitch your boat onto a rocky coast. Presumably also to take a shorter route. One could follow flights of birds to cross the Atlantic, from Europe to Iceland to Greenland to Newfoundland. In the Pacific, one could follow birds and know that a stationary cloud on the horizon meant an island under it. Polynesian navigators could also read the swells and waves, determine in which direction land would lie due to the interference in the wave patterns produced by a land mass.

And then there are the stars; one in particular, the north pole star Polaris. For any given port city, Polaris would always be more or less at a constant altitude angle above the horizon all year at any hour. Latitude hooks, the kamal, and the astrolabe are ancient tools that allowed one to measure the altitude of the Pole Star. So long as your last stage of sailing was due east or west, you could get back home if Polaris was at the same altitude angle as when you left. If you knew the altitude angle of Polaris for your destination, you could sail north or south to pick up the correct Polaris altitude, then 'run down the latitude' until you arrive at the destination (interestingly Polaris was not always the Pole Star). Determining longitude would remain a mystery for many ages until accurate clocks could be made. Techniques used in surveying were adopted for use in navigation, two of which are illustrated on the next page.
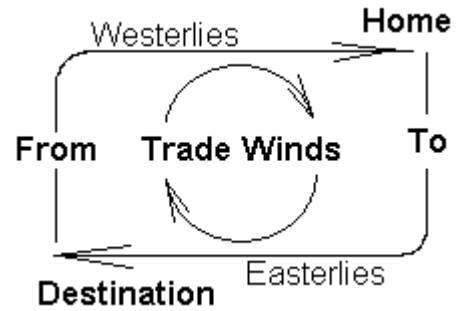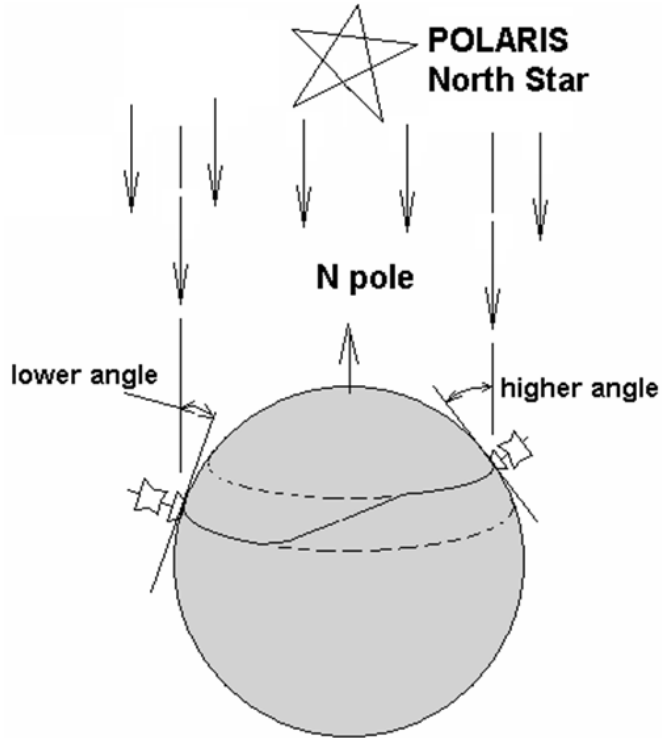
'Running down the latitude' from home to destination, changing latitude where safe to pick up trade winds

Surveying techniques with absolute angles and relative angles
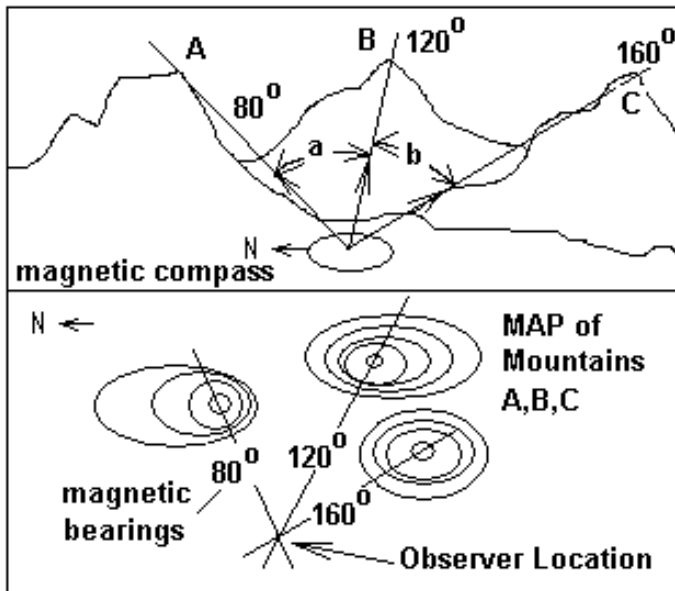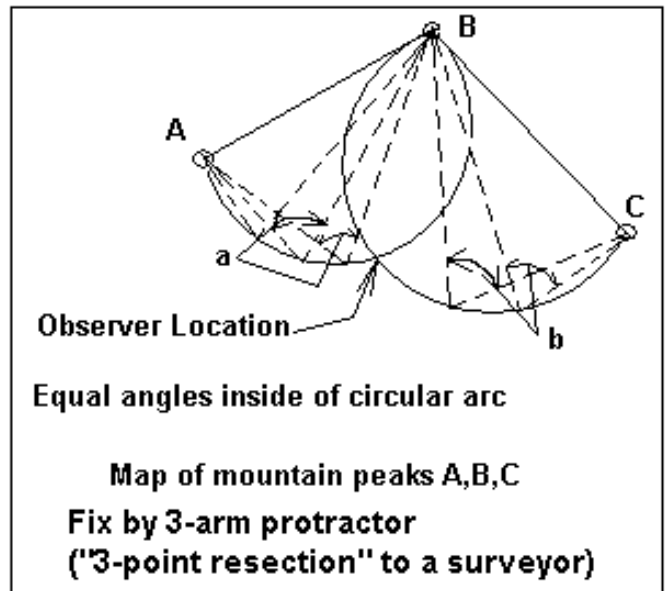
Chapter Two   *Review of Fundamentals*

Orbits

The Earth's orbit about the Sun is a slightly elliptical one, with a mean distance from the Sun equal to 1 AU (AU = Astronomical Unit = 149,597,870. km). This means that the Earth is sometimes a little closer and sometimes a little farther away from the Sun than 1 AU. When it's closer, it is like going downhill where the Earth travels a little faster thru its orbital path. When it's farther away, it is like going uphill where the Earth travels a little slower. If the Earth's orbit were perfectly circular, and was not perturbed by any other body (such as the Moon, Venus, Mars, or Jupiter), in which case the orbital velocity would be unvarying and it could act like a perfect clock. This brings us to the next topic…

Mean Sun

The mean Sun is a fictional Sun, the position of the Sun in the sky if the Earth's axis was not tilted and its orbit were truly circular. We base our clocks on the mean Sun, and so the mean Sun is another way of saying the year-averaged 24 hour clock time. This leads to the situation where the true Sun is up to 16 minutes too fast or 14 minutes too slow from clock reckoning. This time difference between the mean Sun and true Sun is known as the Equation of Time. The Equation of Time at local noon is noted in the Nautical Almanac for each day. For several months at a time, local noon of the true Sun will be faster or slower than clock noon due to the combined effects of Earth's orbital eccentricity and orbital velocity. When we graph the Equation of Time in combination with the Sun's declination angle, we produce a shape known as the Analemma. The definition and significance of solar declination will be explained in a later section.

THE ANALEMMA
Equation of Time
+ Sun fast, minutes — Sun slow, minutes
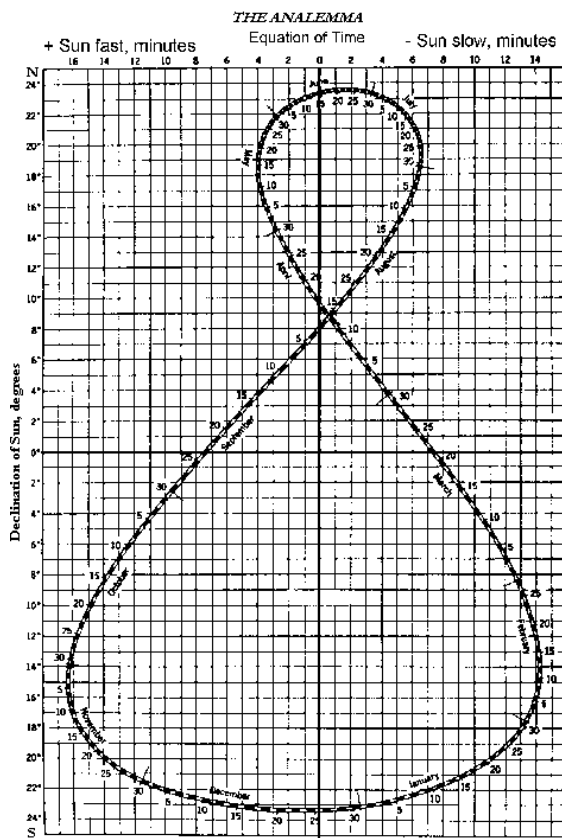
The Analemma: a graphical presentation of the equation of time

Time

        With a sundial to tell us local noon, and the equation of time to tell us the difference between solar and mean noon, a simple clock could always be reset daily. We think we know what we mean when we speak of time, but how to measure it? If we use the Earth as a clock, we could set up a fixed telescope pointing at the sky due south with a vertical hair line in the eyepiece and pick a guide star that will pass across the hairline. After 23.93 hours (a sidereal day, more later) from when the guide star first crossed the hairline, the star will pass again which indicates that the earth has made a complete revolution in inertial space. Mechanical clocks could be reset daily according to observations of these guide stars. A small problem with this reasonable approach is that the Earth's spin rate is not completely steady, nor is the direction of the Earth's spin axis. It was hard to measure, as the Earth was our best clock, until atomic clocks showed that the Earth's rate of rotation is gradually slowing down due mainly to tidal friction, which is a means of momentum transference between the Moon and Earth. Thus we keep fiddling with the definition of time to fit our observations of the heavens. But orbital calculations for planets and lunar positions (ephemeris) must be based on an unvarying absolute time scale. This time scale that astronomers use is called Ephemeris Time. Einstein of course disagrees with an absolute time scale, but it is relative to Earth's orbital speed.

Time Standards for Celestial Navigation

        Universal Time (UT, solar mean time, GMT)
This standard keeps and resets time according to the mean motion of the Sun across the sky over Greenwich England, the prime meridian, (also known as Greenwich Mean Time GMT). UT is noted on a 24-hour scale, like military time. The data in the nautical almanac is based on UT.

Universal Time Coordinated (UTC)

        This is the basis of short wave radio broadcasts from WWV in Fort Collins Colorado and WWVH in Hawaii (2.5, 5,10,15, 20 MHz). It is also on a 24-hour scale. It is synchronized with International Atomic Time, but can be an integral number of seconds off in order to be coordinated with UT such that it is no more than 0.9 seconds different from UT. Initial calibration errors when the atomic second was being defined in the late 1950's, along with the gradual slowing of the Earth's rotation, we find ourselves with one more second of atomic time per year than a current solar year. A leap second is added usually in the last minute of December or June to be within the 0.9 seconds of UT. UTC is the time that you will use for celestial navigation using the nautical almanac, even though strictly speaking UT is the proper input to

the tables. The radio time ticks are more accessible, and 0.9 seconds is well within reasonable error.

Sidereal Year, Solar Year, Sidereal Day, Solar Day

There are 365.256 solar days in a sidereal year, the Earth's orbital period with respect to an inertially fixed reference axis (fixed in the 'ether' of space, or in actuality with respect to very distant stars). But due to the backward clockwise precession drift of the equinox (the Earth orbits counterclockwise as viewed above the North Pole), our solar year (also referred to as tropical year) catches up faster at 365.242 solar days. We base the calendar on this number as it is tied into the seasons. With 360 degrees in a complete circle, coincidentally (or not), that's approximately 1 degree of orbital motion per day (360 degrees/365.242 days). That means inertially the Earth really turns about 361 degrees every 24 hours in order to catch up with the Sun due to orbital motion. That is our common solar (synodic) day of 24 hours. However, the true inertial period of rotation is the time it takes the Earth to spin in 360 degrees using say, the fixed stars as a guide clock. That is a sidereal day, 23.93447 hours (~ 24 x 360/361). The position of the stars can be measured as elapsed time from when the celestial prime meridian passed, and that number reduced to degrees of celestial longitude (SHA) due to the known rotational period of the Earth, a sidereal day. As a side note, this system of sidereal hour angle SHA is the negative of what an astronomer uses, which is right ascension (RA).
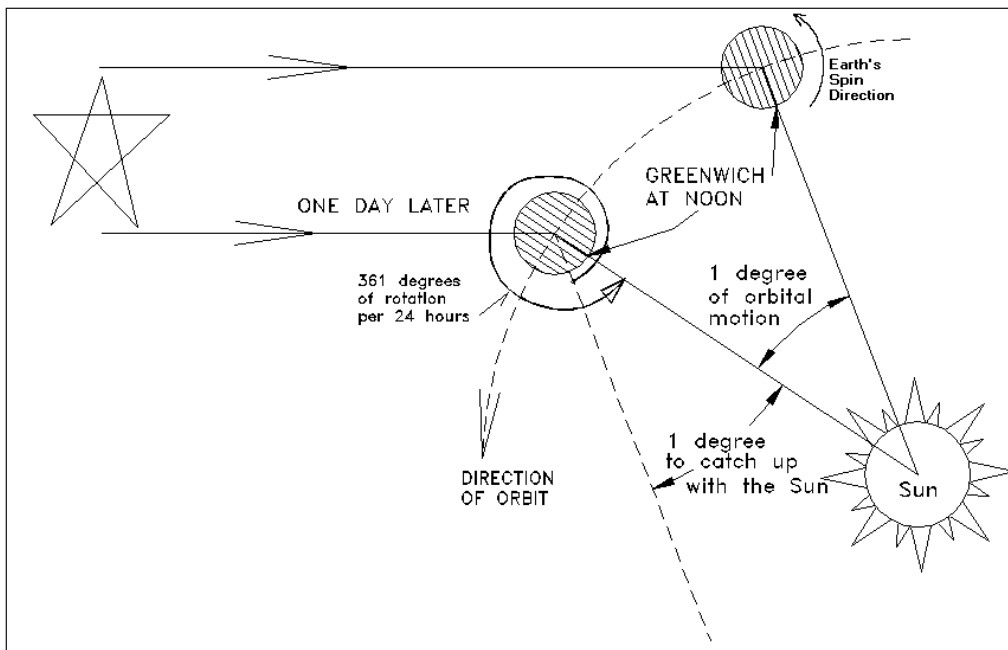
The difference between a Sidereal day and a Solar day

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Latitude and Longitude

I will not say much on this, other than bringing your attention to the illustration, which show longitude lines individually, latitude lines individually, and the combination of the two. This gives us a grid pattern by which unique locations can be associated to the spherical map using a longitude coordinate and a latitude coordinate. The prime N-S longitude meridian (the zero longitude) has been designated as passing thru the old royal observatory in Greenwich England (established 1884). East of Greenwich is positive longitude, and west of Greenwich is negative longitude. North latitude coordinates are positive numbers, south latitude coordinates are negative.
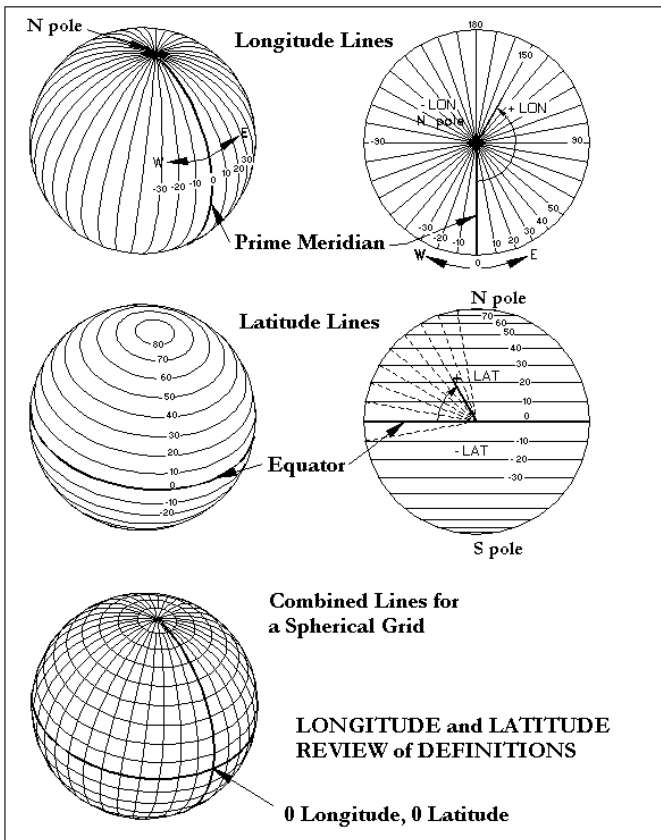
Maps and Charts

The most common chart type is the modern *Mercator projection*, which is a mathematically modified version of the original cylindrical projection. On this type of chart, for small areas only in the map's origin, true shapes are preserved, a property known as *conformality*. Straight line courses plotted on a Mercator map have the property of maintaining the same bearing from true north all along the line, and is known as a *rumb line*. This is a great aid to navigators, as the course can be a fixed bearing between waypoints.

If you look at a globe and stretch a string from point A to point B, the path on the globe is a *great circle* and it constitutes the shortest distance between two points on a sphere. The unfortunate characteristic of a great circle path is that the bearing relative to north changes along the length of the path, most annoying. On a Mercator map, a great circle course will have the appearance of an arc, and not look like the shortest distance. In fact, a rumb line course mapped onto a sphere will eventually spiral around like a clock spring until it terminates at either the N or S pole, known mathematically as a loxodrome.

Chapter Three   *Celestial Navigation Concepts*

There are three common elements to celestial navigation, whether one is floating in space, or floating on the ocean. They are; 1) <u>knowledge</u> of the positions of heavenly bodies with respect to time, 2) <u>measurement</u> of the time of observation, and 3) angular <u>measurements</u> (altitudes) between heavenly objects and a known reference. The reference can be another heavenly object, or in the case of marine navigation, the horizon. If one only has part of the required 3 elements, then only a partial navigational solution will result. In 3 dimensions, one will need 3 independent measurements to establish a 3-D position fix. Conveniently, the Earth is more or less a sphere, which allows an ingeniously simple technique to be employed. The Earth, being a sphere, means we already know one surface that we must be on. That being the case, all we need are 2 measurements to acquire our fixed position on the surface.

Listed below is the stepwise Generalized Celestial Navigation Procedure:
1) Estimate the current position
2) Measure altitude angles of <u>identified</u> heavenly bodies
3) Measure time at observation with a chronometer
4) Make corrections to measurements
5) Look up tabulated ephemeris data in the nautical almanac
6) Employ error-reduction techniques
7) Employ a calculation algorithm
8) Map the results, determine the positional fix

The 4 basic tools used are the sextant, chronometer, nautical almanac, and calculator (in lieu of pre-calculated tabulated solutions).

In this book and in most celestial navigation texts, altitudes (elevation angle above the horizon) of the observed heavenly object s are designated with these variables:
Hs = the raw angle measurement reported by the sextant's scale.
Ha = the apparent altitude, when instrument errors and horizon errors are accounted for.
Ho = the true observed altitude, correcting Ha for atmospheric refraction and geometric viewing errors (parallax) associated with the heavenly object.

Sextant, Chronometer (time piece), Nautical Almanac, and a Calculator

Short hand notation for angles

For degrees we use ^{0}, and for arc-minutes we use $'$
For example, 20 degrees 45 arc-minutes is expressed as $20^0\ 45'$
In decimal degrees that would be $20 + 45/60 = 20.7500^0$

Geographical Position (GP)

The geographical position of a heavenly object is the spot on the Earth's surface where an observer would see the object directly overhead, the zenith point. You can think of it as where a line connecting the center of the Earth and the center of the heavenly object intersects the Earth's surface. Since the Earth is spinning on its axis, the GP is always changing; even for Polaris since it is not exactly on the axis (it is close…)

Circles of Position (COP)

Every heavenly object seen from the Earth can be thought of as shining a spotlight on the Earth's surface. This spotlight, in turn, cast concentric circles on the Earth's surface about the GP. At a given moment anybody anywhere on a particular circle will observe the exact same altitude for the object in question. These are also known as circles of constant altitude.
For the most part, stars are so far away that their light across the solar system is parallel. The Sun is sufficiently far away that light from any point on the Sun's disk will be more or less parallel across the face of the Earth. Not so for the Moon.

DISTANT OBJECT CENTER

CENTER-to-CENTER LINE

PARALLEL RAYS OF LIGHT

GEOGRAPHICAL POSITION AT ZENITH POINT

ALTITUDE A

ALTITUDE B

BOAT B

BOAT A

LOCAL HORIZON

SAME AS ALTITUDE B

EARTH CENTER

SAME AS ALTITUDE A

CIRCLES OF CONSTANT ALTITUDE

CIRCLES OF CONSTANT ALTITUDE

CIRCLES OF CONSTANT ALTITUDE FOR A DISTANT OBJECT
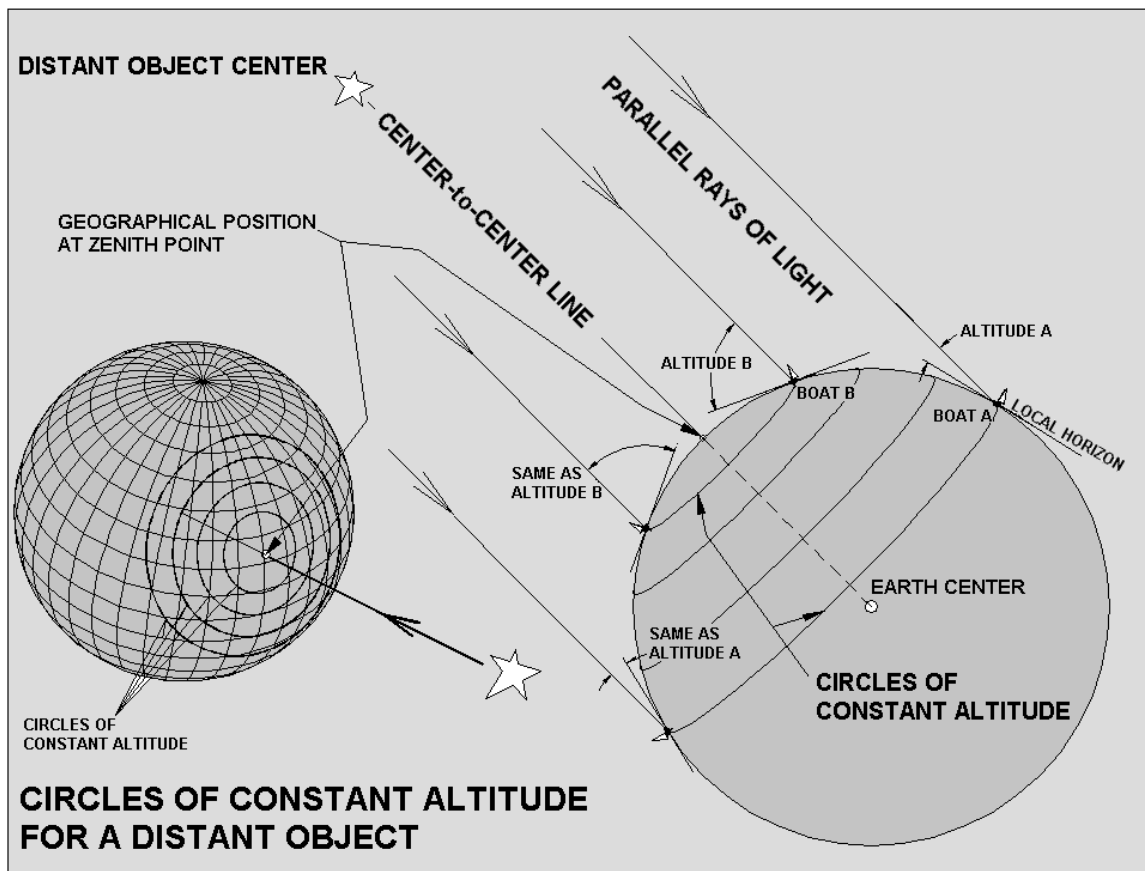
Parallax

This is a geometrical error that near-by heavenly objects, namely the Moon, are guilty of. Instead of a spotlight of parallel light, a near-by object casts more of a conical floodlight. The reason why parallax matters to us is because in the nautical almanac, the center-to-center line direction from the Earth to the heavenly object is what is tabulated. The particular cone angle is not tabulated, and needs to be calculated and added to the observed altitude to make an apples-to-apples comparison to the information in the almanac. The Moon's parallax can be almost 1 degree, and needs to be accounted for. The parallax can be calculated easily, if we know how far away the heavenly object is (which we do). From the illustration, it should be apparent that the parallax is a function of the altitude measurement. It is a constant number for anyone on a particular circle of constant altitude. The particular parallax angle correction corresponding to the particular altitude is known as *parallax-in-altitude* PA. The maximum parallax possible is when the altitude is equal to zero (moonrise, moonset) and is designated as the *horizontal parallax* HP.

CIRCLES OF CONSTANT ALTITUDE FOR A NEAR OBJECT
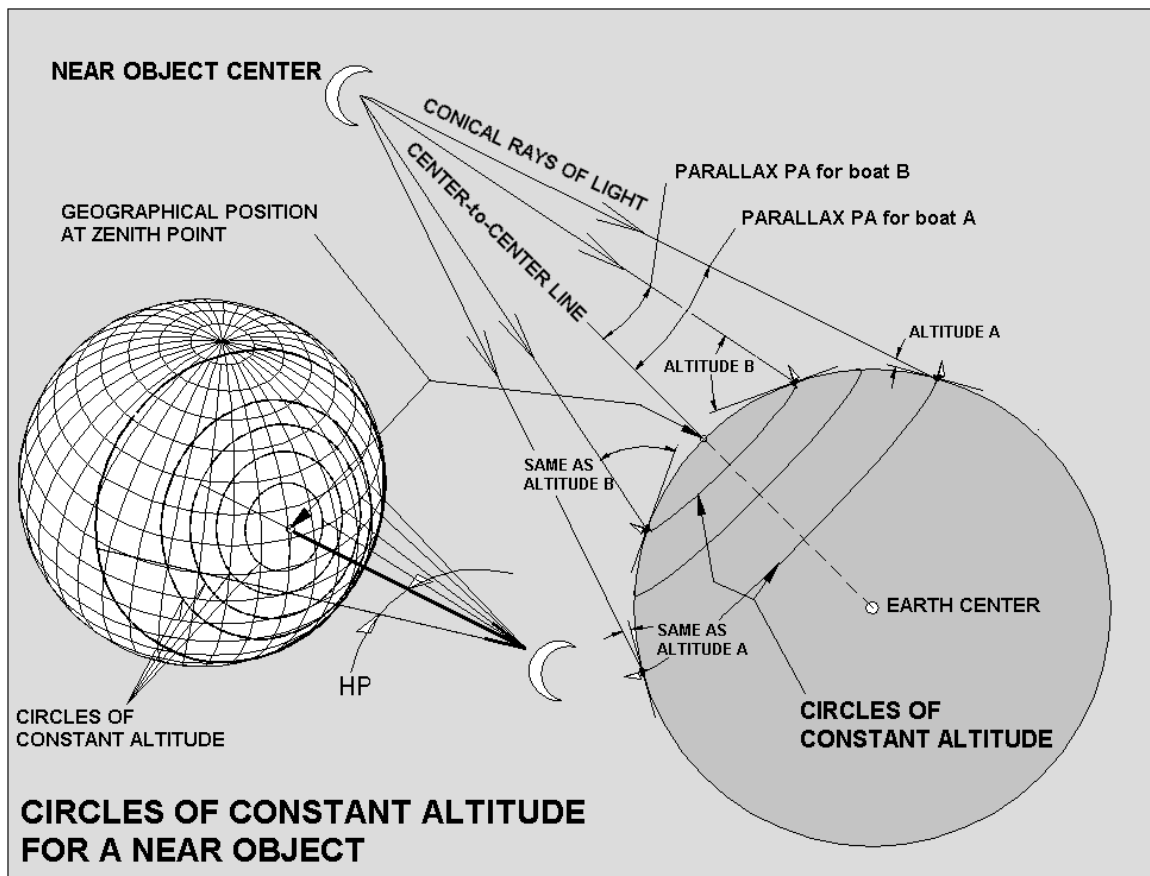
Line of Position (LOP)

Circles of Position can have radii thousands of miles across, and in the small vicinity of our estimated location on the map, the arc looks like a line, and so we draw it as a line tangent to the circle of constant altitude. This line is necessarily perpendicular to the azimuth direction of the heavenly object. One could be anywhere (within reason) on that line and measure the same altitude to the heavenly object.
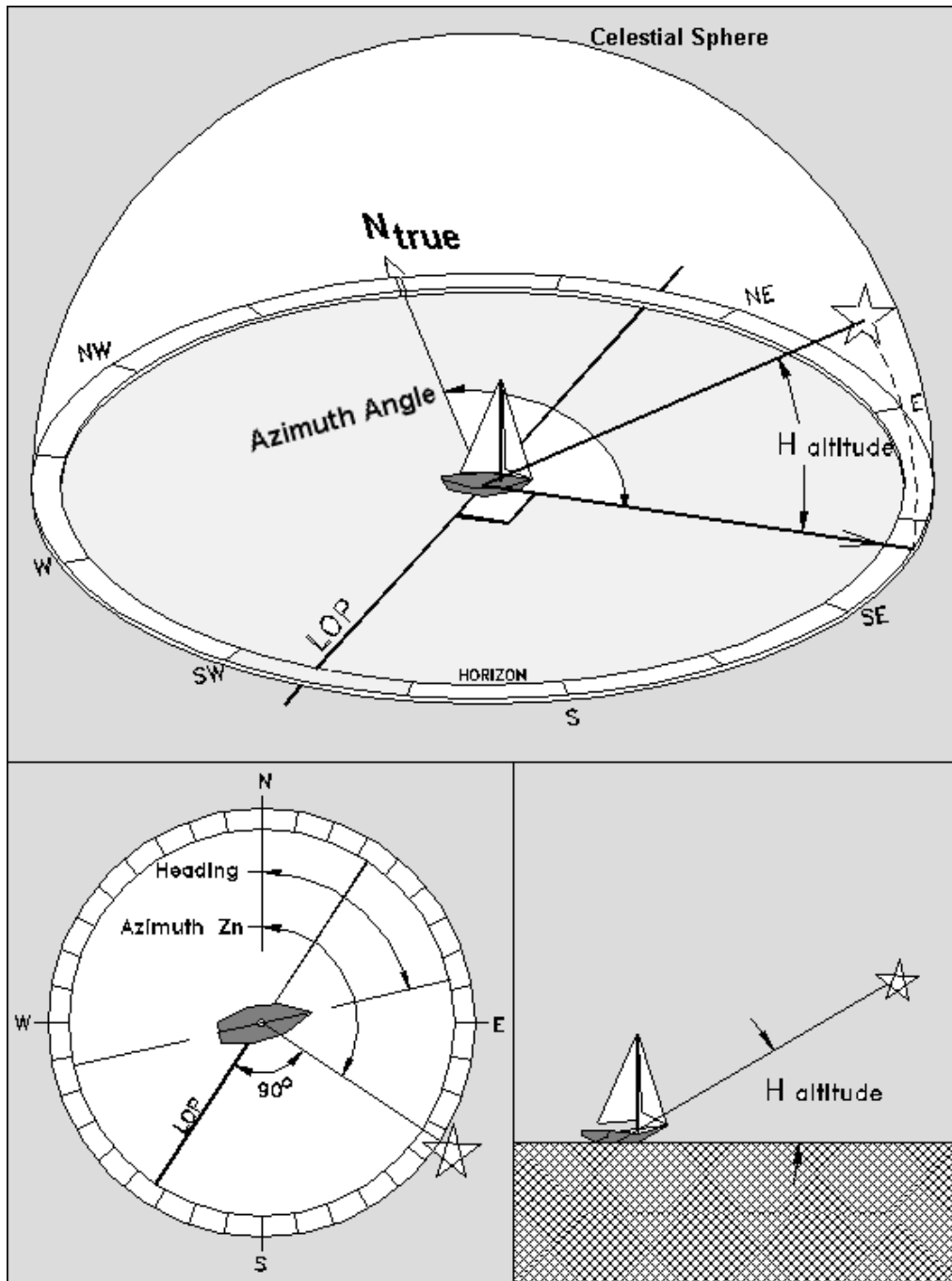
Navigational Fix

To obtain a 'fix', a unique latitude and longitude location, we will need two heavenly objects to observe. Reducing the measurements to 2 LOPs, the spot where it crosses the 1st line of position is our pin-point location on the map, the navigational fix. This is assuming you are stationary for both observations. If you are underway and moving between observations, then the first observation will require a '*running fix*' correction. See the illustration of the navigational fix to see the two possibilities with overlapping circles of constant altitude. The circles intersect in two places, and the only way to be on both circles in the same place is to be on one of the two intersections. Since we know the azimuth directions of the observations, the one true location becomes obvious. Measurement errors of angle and time put a box of uncertainty around that pinpoint location, and is called the *error box*.

We could of course measure the same heavenly object twice, but at different times of the day to achieve the same end. This will produce two different circles of constant altitude, and where they intersect is the fix, providing you stay put. If you're not, then running fix corrections can be applied here as well. In fact, this is how navigating with the Sun is done while underway with observations in the morning, noon, and afternoon.

More often than not, to obtain a reliable fix, the navigator will be using 6 or more heavenly objects in order to minimize errors. Stars or planets can be mistakenly identified, and if the navigator only has 2 heavenly objects and one is a mistake, he/she may find themselves in the middle of New Jersey instead of the middle of the Atlantic. It is improbable that the navigator will misidentify the Sun or Moon (one would hope…), but measurement errors still need to be minimized. The two measurements of time and altitude contain *random errors* and *systematic errors*. One can also have calculation errors and misidentification errors, correction errors, not to mention that you can simply read the wrong numbers from the almanac.

The random errors in measurement are minimized by taking multiple 'shots' of the same object (~3) at approximately one minute intervals, and averaging the results in the hope that the random errors will have averaged out to zero. Systematic errors (constant value errors that are there all the time) such as a misaligned sextant, clocks that have drifted off the true time, or atmospheric optical effects different from 'normal' viewing conditions all need to be minimized with proper technique and attention to details, which will be discussed later. Another source of systematic error is your own 'personal error', your consistent mistaken technique. Perhaps you are always reading a smaller angle, or you are always 1 second slow in the clock reading. This will require a 'personal correction'.

THE NAVIGATIONAL FIX
with 2 CELESTIAL OBJECTS

Azimuth Direction
of Object #2

You are here (see inset for details)

OBJECT #2

Azimuth Direction
of Object #1

Geographical Position
of Object #2

Circles of Constant Altitude

Geographical Position
of Object #1

OBJECT #1

#2

N TRUE

NE

#1

LOP #2

NW

E

H₂

H₁

Azimuth Line #2

Azimuth Line #1

W

LOP #1

SE

SW

HORIZON

S
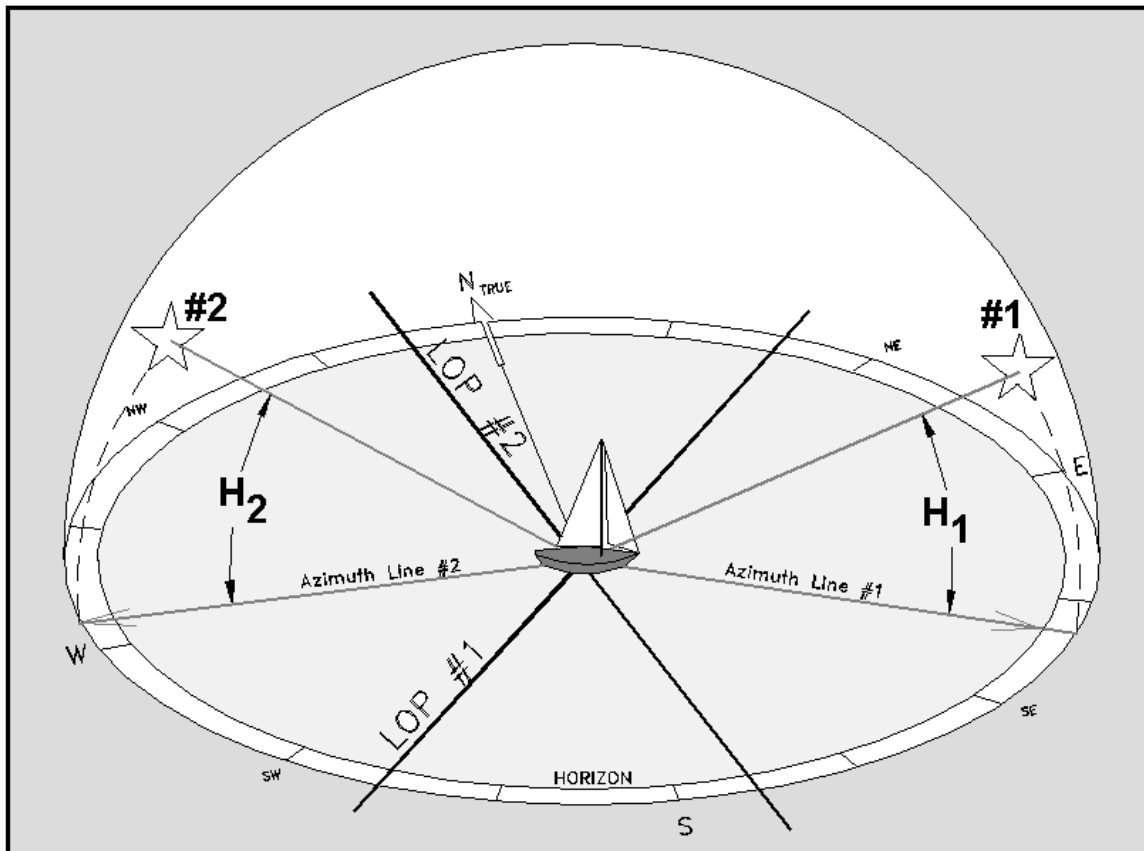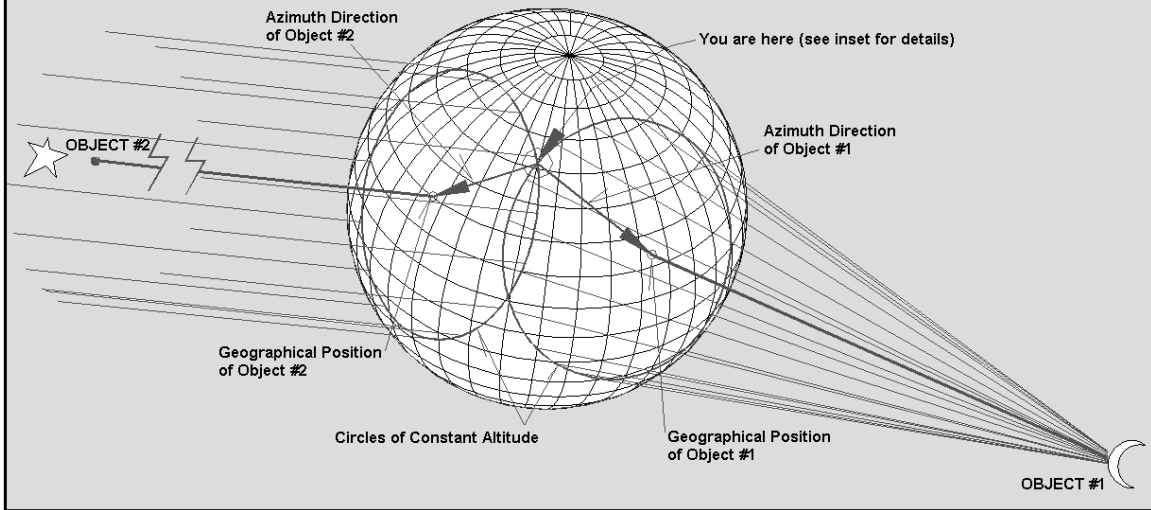
Surfaces of Position (SOP)

If you were floating in space, you could measure the angle between the Sun and a known star. There will exist a conical surface with the apex in the Sun's center with the axis of the cone pointing in the star's direction whereby any observer on that conical surface will measure the exact same angle. This is a Surface of Position, where this one measurement tells you only that you are somewhere on the surface of this imaginary cone. Make another measurement to a second star, and you get a second cone, which intersects the first one along two lines. Now, the only way to be on both cones at the same time is to be on either of those 2 intersection lines. Make a third measurement between the Sun and a planet, and you will create a football shaped Surface of Position, with the ends of the football centered on the Sun and the planet (see pg. 8). This third SOP intersects one of the two lines at one point. That is your position in 3-D.

Notice that if the football shape enlarges to infinity, the end points locally resemble cones. This is what star cones 1&2 actually are. If you used a third star instead of a planet, you would create another pair of intersection lines, one of which will be collinear with one of the 1st pair. It will not get you a point. You need to have a nearby object for the final fix. The football shape is merely the circular arc method revolved about an axis to create a 3-D football surface.

1st Observation, angle A
from star #1 to Sun
STAR #1
A
Sun

2nd Observation, angle B
from star #2 to Sun
STAR #2
Sun
B
Sun
C
3rd Observation, angle C
from Sun to Planet
Planet

Merge the 2 cones together to produce
2 Lines of Position (LOP)
Sun

Merge the 2 lines of position and the
'football' shape to intersect the position fix
Sun
Sun
Observer
Position Fix
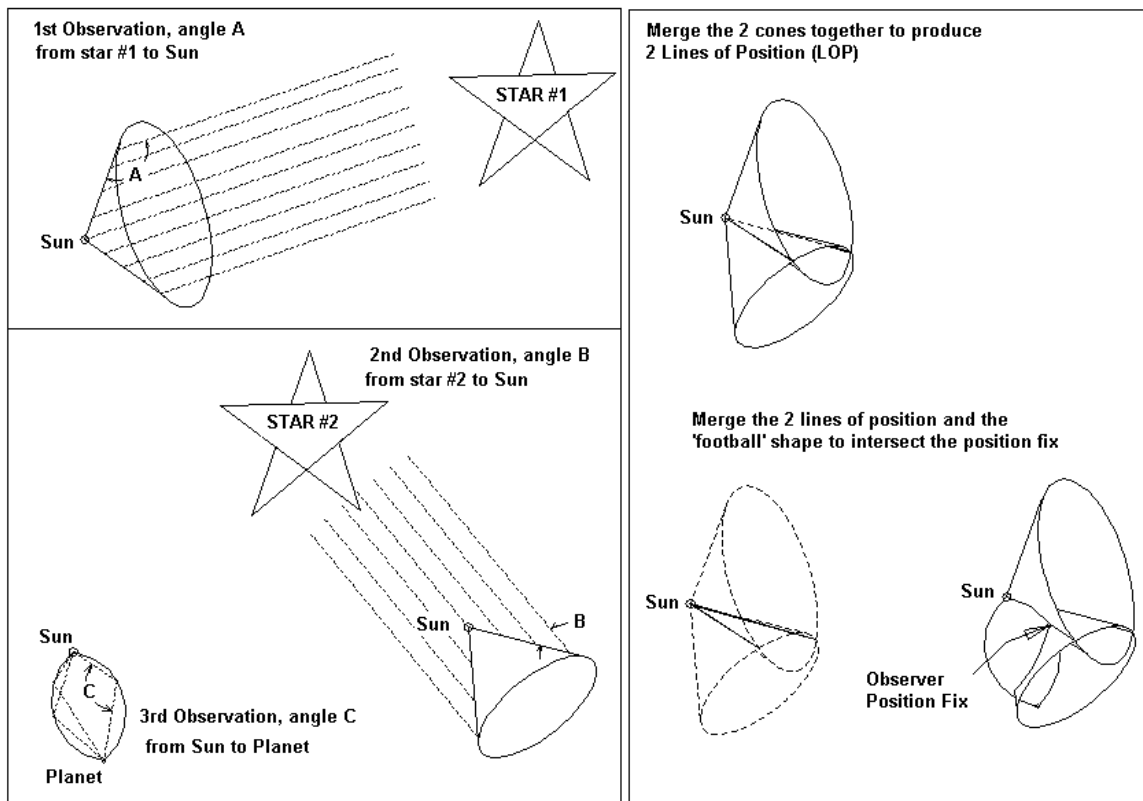
Celestial Sphere

The *celestial sphere* is our star map. It is not a physical sphere like the Earth's surface. It is a construction of convenience. The stars do have a 3-dimensional location in space, but for the purposes of navigation we mostly need to know only their direction in the sky. For stars, their distance is so great that their dim light across the solar system is more or less parallel. With that thought, we can construct a transparent sphere which is like a giant bubble centered over the Earth's center where the fixed stars are mapped, painting the stars, Sun and our solar system planets on the inside of this sphere like a planetarium. We are on the inside of the bubble looking out. The celestial sphere has an equatorial plane and poles just like the Earth. In fact, we define the *celestial poles* to be an extension of Earth's poles, and the two equatorial planes are virtually the same. It just does not spin. It is fixed in space while the Earth rotates inside it.

In our lifetimes, the stars are more or less fixed in inertial space. Their very slow movement is called *proper motion*. However, the apparent location of a star changes slightly on the star map due to *precession* and *nutation* of the Earth's axis, as well as *annual aberration*. That is, the Earth's spin axis does not constantly point in the same direction. We usually think of the North Pole axis always pointing at Polaris, the North Star (it's currently 41' off from the pole). It wiggles (nutates) around it now, but in 10000 years it will point and wiggle about Deneb. However, 5000 years ago it pointed at Thuban and was used by the ancient Egyptians as the Pole Star! The Earth wobbles (precession) in a cone-like shape just like a spinning top, cycling once every 25800 years. We know the cone angle to be the same as the 23.44 degree tilt angle of the Earth's axis, but even that tilt angle wiggles (nutates) up and down about 0.15 arcminutes. There are two periods of nutation, the quickest equal to ½ year due to the Sun's influence, and the slowest (but largest) lasting 18.61 years due to the Moon's precession (wobbling) orbital plane tugging on the earth.

Aberration is the optical tilting of a star's apparent position due to the relative velocity of the earth vs. the speed of light. Think of light as a stream of particles like rain (photons) speeding along at 299,792 kilometers/s. The Earth is traveling at a mean orbital velocity of 29.77 kilometers/s. When you run in the rain, the direction of the rain seems to tilt forward. The same effect is true of light, with the least effect from stars near the ecliptic plane, and the most effect from stars with the highest elevation from the ecliptic plane. This effect can be as great as 20.5 arc-seconds (3600 x arcTan(29.77/299792)).

The *ecliptic plane* (Earth's orbital plane at a given reference date, or *epoch*) mapped onto the celestial sphere is where you will also see the constellations of

the *zodiac* mapped. These are the constellations that we see planets traverse across in the night sky, and therefore got special attention from the ancients. Instead of describing the location of a star on the celestial sphere map with longitude and latitude, it is referred to as Sidereal Hour Angle (SHA) and declination (DEC) respectively. Sidereal Hour Angle is a celestial version of west longitude, and declination is a celestial version of latitude. But this map needs a reference, a zero point where its *celestial prime meridian* and celestial equator intersect. That point just happens to be where the Sun is located on the celestial sphere during the spring (*vernal*) equinox, and is known as the *Point of Aries*. It is the point of intersection between the mean equatorial plane and the ecliptic plane. Since the Earth's axis wiggles and wobbles, a reference mean location for the equatorial plane is used. Due to precession of the Earth's axis, that point is now in the zodiacal constellation of Pisces, but we say Aries for nostalgia. That point will travel westward to the right towards Aquarius thru the zodiac an average of 50.3 arcseconds per year due to the 25800 year precession cycle. Fortunately, all of these slight variations are accounted for in the tables of the nautical and astronomical almanacs.

Local Celestial Sphere

This is the celestial sphere as referenced by a local observer at the center with the true horizon as the equator. *Zenith* is straight up, *nadir* is straight down. The local meridian circle runs from north to zenith to south. The prime vertical circle runs from east to zenith to west.
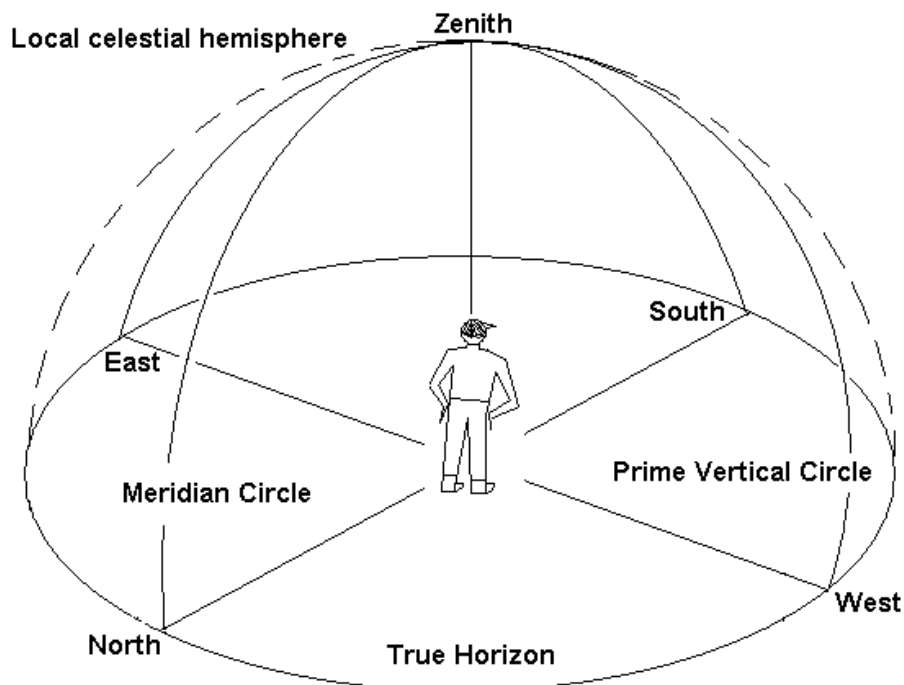
Local celestial sphere for a ground observer

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EARTH'S PRECESSION and NUTATION

23.44 °

Mean Precession motion is clockwise in a conical shape; 25800 years per cycle

Wiggley motion is Nutation

Basic nutation periods range from 0.5 years to 18.6 years

Earth's Daily Rotation counter clockwise (CCW)

The Celestial Sphere

Celestial North Pole

Sidereal Hour Angle

CELESTIAL PRIME MERIDIAN
Zero Hour Angle

Celestial Equator

12hr Hour Angle

SPRING EQUINOX
Point of Aries

Aries

Pisces

Aquarius

SHA

EQUATORIAL PLANE

Capricorn

Declination

Tarus

23.44°

September

Sagitarius

Gemini

N

December

June

ECLIPTIC PLANE
(ORBIT PLANE)

S

March

Cancer

ZODIAC

Scorpio

Leo

Libra

Virgo

FALL EQUINOX

Celestial South Pole

Greenwich Hour Angle GHA

The *Greenwich Hour Angle* (GHA) of a heavenly object, is the west longitude of that object at a given instant in time relative to the Earth's prime meridian. The Sun's GHA is nominally zero at noon over Greenwich, but due to the slight eccentricity of Earth's orbit (mean vs. true sun) it can vary up to 4 degrees. GHA can refer to any heavenly object that you are using for navigation, including the position of the celestial prime meridian, the point of Aries.

Bird's-eye view above the North Pole
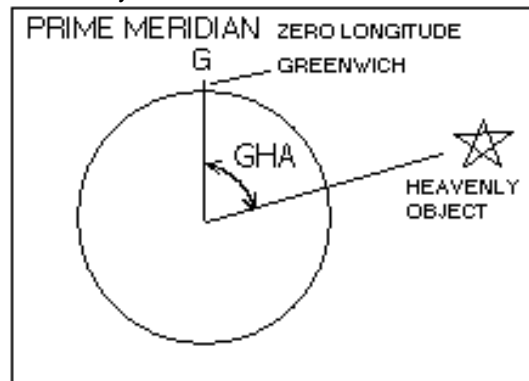
Greenwich Hour Angle of Aries GHA$_{Aries}$ (or GHAγ)

The point of Aries is essentially the zero longitude and latitude of the celestial sphere where the stars are mapped. The sun, moon, and planets move across this map continuously during the year. SHA and declination relate the position of a star in the star map, and **GHA$_{Aries}$** relates the star map to the Earth map. GHA$_{Aries}$ is the position of the zero longitude of the star map, relative to Greenwich zero longitude, which varies continuously with time because of Earth's rotation. The relationship for a star is thus:

GHA = GHA$_{Aries}$ + SHA = the Greenwich hour angle of a star. The declination (celestial latitude) of the star needs no 'translation' as it remains the same in the Earth map as in the star map.

Bird's-eye view above the North Pole

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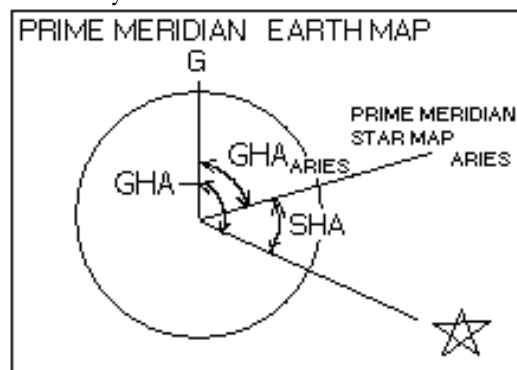
Local Hour Angle LHA

The *Local Hour Angle* (LHA) is the west longitude direction angle of a heavenly object relative to a local observer's longitude (not Greenwich). This leads to the relationship:

LHA = GHA + East Longitude Observer, or
LHA = GHA - West Longitude Observer

If the calculated value of LHA > 360, then LHA = LHA$_{CALCULATED}$ - 360

Bird's-eye view above the North Pole
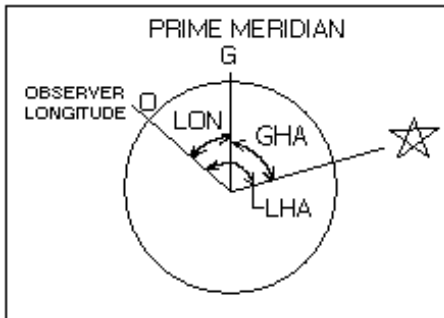
When we are speaking of the Sun, a pre-meridian passage (negative LHA or 180<LHA < 360) means that it is still morning. A post-meridian passage (positive LHA, or 0<LHA<180) means that it is literally after noon.

At exact local noon, LHA = 0

Declination DEC

As stated earlier, the *declination* of an object is the celestial version of latitude measured on the celestial sphere star-map. Due to the tilt of the Earth's axis of 23.44 degrees, the sun and planets change their declinations on the celestial sphere continuously during the year. The stars do not except for precession and nutation effects. The Sun's declination follows nearly a perfect sine wave where over the course of 365.24 days it varies northwards 23.44 degrees and southwards –23.44 degrees. This is a crucial piece of information for the determination of latitude using the Sun.
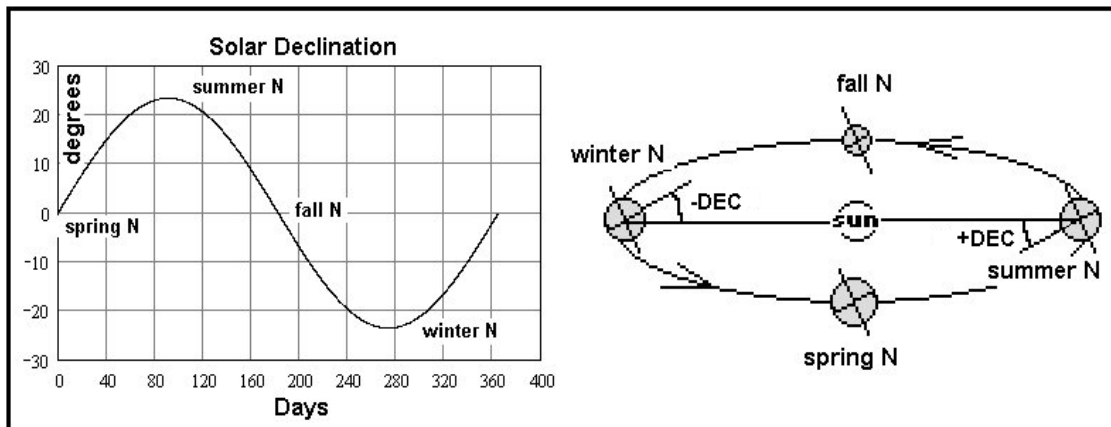
As one can see, maximum declination occurs with the summer solstice which has the longest hours of daily sun, the minimum declination with the winter solstice having the shortest hours of sun, and the spring and fall equinox ("equal night") having equal day and night times corresponding to zero solar declination. During the equinoxes, the sun will rise directly from the east and set directly in the west. At 40 degrees latitude, there are 6 more hours of daylight in the summer as compared to the winter.
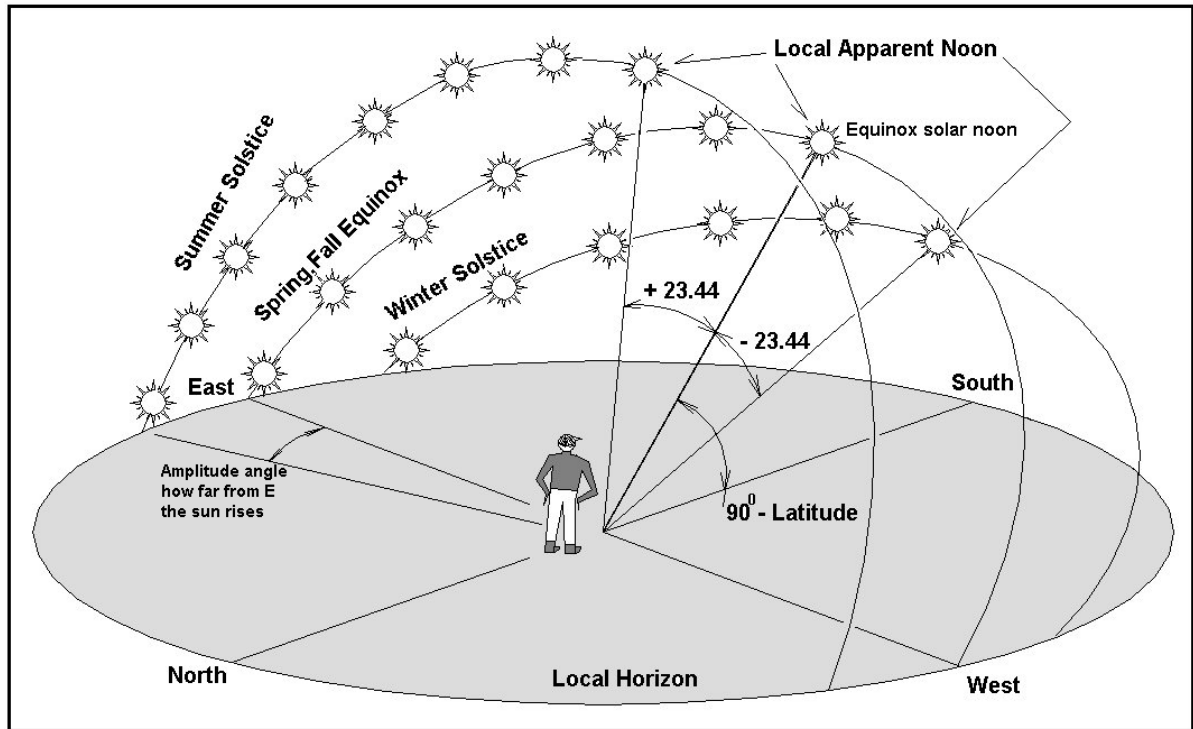
Solar declination effects seen by an observer on the ground varying seasonally

Sign Convention

We should digress momentarily to establish the proper signs for numbers, which make the mathematics consistent and unambiguous.

For Declination: North is positive (+)   South is negative (-)
For Latitude: North is positive (+)   South is negative (-)
For Longitude: East is positive (+)   West is negative (-)
For GHA, it is a positive number between 0 and 360 degrees westward
For LHA, it is positive westwards (post meridian passage) $0 < LHA < 180$, and negative eastwards (pre meridian passage) $-180 < LHA < 0$, or $180 < LHA < 360$
For observed altitude, **Ho**, above the horizon is positive (+)
Also if you are not aware, in an equation A·B means A multiplied by B (A**x**B).

Concepts in Latitude

The simplest example to illustrate how latitude is determined is to consider Polaris, the North Star. Now Polaris is not exactly on the north celestial pole, but close enough for our intuition to work here. If we were sitting on Earth's north pole (avoiding polar bears), we would observe that Polaris would be directly overhead, at the zenith point. Relative to the horizon, it would have an altitude of approximately 90 degrees of angle. Our latitude at the North Pole coincidentally is also 90 degrees. If now instead we were sweating somewhere on the equator on a hill in Ecuador at night, we would see Polaris just on the northern horizon. The altitude relative to the horizon would be approximately zero. Coincidentally, the latitude on the equator is zero. To see why this is not really a coincidence, see the illustration to understand the geometry involved. We could say generally that the observed altitude of Polaris is equal to the latitude of the observer (actually small corrections need to be made since Polaris is slightly off center from the pole). Also note that the declination of Polaris in the celestial sphere is about 90 degrees. We can generalize the matter by taking into account the declination of any particular star, as shown in the illustration. Such a star can be the Sun, and if we know the declination for every hour of the year, we can wait until the Sun is at its meridian passage (local apparent noon LAN) to make an altitude measurement Ho. The Latitude is then 90 + DEC - Ho.

In those rare occasions when a celestial object reaches 90 degrees altitude, your latitude is equal to the object's declination at that moment in time. That is a good emergency navigation trick, and sometimes this happens with a noon sighting or with stars when the conditions are just right.

CONCEPTS IN LATITUDE DETERMINATION
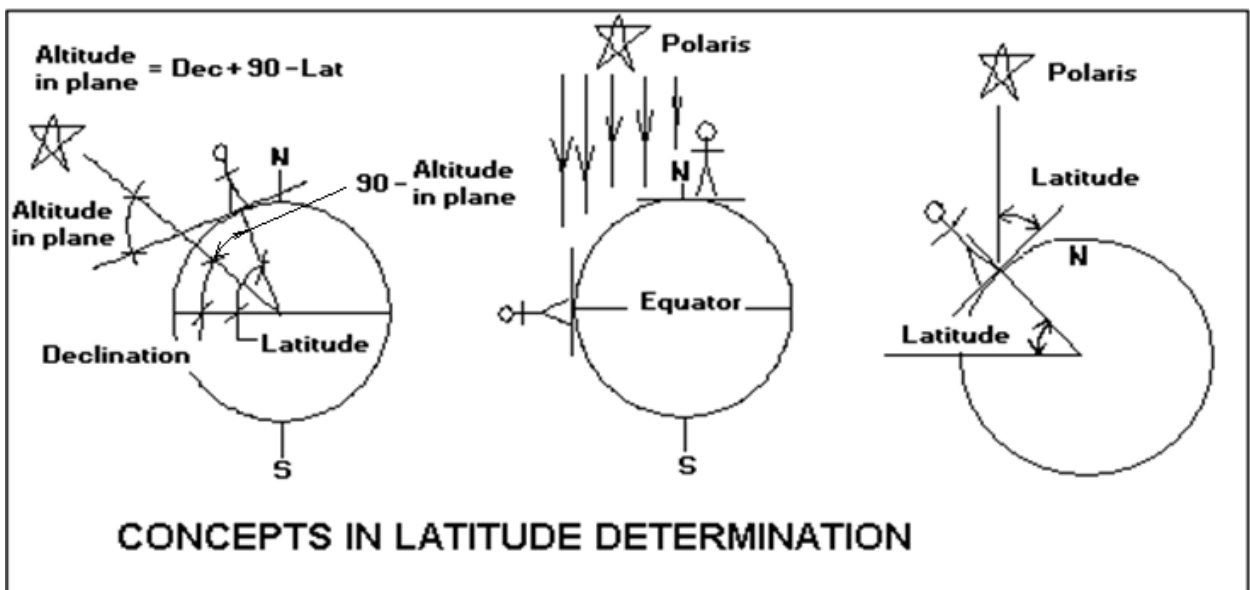
Concepts in Longitude

If we think of a car traveling at 60 mph, in 2 hours it will have traveled 120 miles (60 · 2). To determine distance, all we needed was knowledge of the speed, and a clock. For a rotating object, it is the same. If we know the rotational speed, say ¼ revolutions per minute (RPM), and we have a stopwatch, in 2 minutes it should have rotated ½ revolution(0.25 · 2), or 180 degrees(0.25 · 2 · 360 degrees per rev). Now let's think of the Earth. It rotates once in 24 hours with respect to the position of the mean Sun in the sky. That's 360 degrees in 24 hours, or 15 degrees per hour (360/24). If a person on the Earth observes the Sun passing across the local N-S meridian line (in other words, local noon), and observes the time to be 15:00 UT, that's 3 hours past noon in Greenwich. You will recall, UT is based on the time in Greenwich, zero longitude. The difference in angle between the observer and Greenwich, is 15 deg/hour x 3 hours = 45 degrees of longitude in the westward direction. This is why the chronometer needs to be synchronized with Greenwich time, so the observer can determine the difference in angle (longitude) with respect to the prime meridian (zero longitude). This idea was noted as early as 1530 by the Flemish professor Gemma Frisius. Pendulum clocks were not suitable for the motion of ships, and it was John Harrison in 1735 that made the first semi portable clock, with its 'grasshopper' escapement and twin balance-arm oscillator. What a contraption! But, it was the start of marine chronometers that could take the rocking and rolling of a ship and not lose a beat.

It is no coincidence that along a *great arc* on the Earth (such as the equator), one minute of arc (1/60 degree) corresponds to one *nautical mile* (n mi) of distance. One nautical mile is equal to 1.15 statute miles. The Earth's circumference is then equal to 21600 n mi (1nm per arcmin x 60 arcmin per deg x 360 deg per full circle). The maximum surface speed of rotation for Sun observations will occur along the equator at 15 n mi per minute of time (21600 n mi per day/(24hr per day x 60 min per hr)). This is also equivalent to ¼ n mi per second of time. It is easy to see now how a time error (either the clock is off or the time is read wrong) can put the longitude determination way off. In mid-latitudes, a time error of 60 seconds will put the longitude off by 10 n mi.

You get the general picture, but actually the true position of the Sun does not correspond with clock time as we have already described earlier. It is a little off due to Earth's elliptical orbit.

The upshot of all this explanation is that to know longitude, one then needs to have a clock set to the time in Greenwich England.

Traditional Noon Sighting

The *noon sighting* is an old way of determining latitude and (with misgivings) longitude, as the azimuth is unambiguously known as either due south or due north. The method has certain steps to maintain accuracy. Here, the trigonometry disappears and reduces down to mere arithmetic. The technique is to predict approximate *local apparent noon* (LAN) for your estimated longitude from *dead reckoning* navigation. Take sightings with your sextant several minutes before LAN, and with a sighting every minute, capture the highest point in the sky that the Sun traveled plus some sightings after meridian passage. You make corrections to obtain the true altitudes, and plot this information as true altitude versus time. From the plot you can smooth the curve and determine the highest point (Ho_{noon}) and estimate the time of LAN to within several minutes or better of Universal Time (~10-20 n miles of longitude error). Using the nautical almanac, obtain the GHA and declination of the Sun (DEC) at the time of LAN. Remember the sign convention and apply it. We will now make a distinction regarding the direction of meridian passage, whether the sun peaked in the south or in the north, by introducing a new variable $Sign_{noon}$. In keeping with the consistent sign convention, when the meridian passage is northwards such as commonly occurs in S. latitudes, the value of $Sign_{noon}$ is +1. When the meridian passage is southwards such as commonly occurs in N. latitudes, the value of $Sign_{noon}$ is -1. Thus:

$$Latitude = \; Sign_{noon} \cdot Ho_{noon} + DEC + 90$$

If this calculated latitude is greater than 90 degrees, then subtract 180 from it.
If $Sign_{noon} \cdot Ho_{noon} + DEC$ is equal to zero, then you are exactly on either the north or south pole. If you don't know which pole you're on then you should have stayed home.

This equation works whether you are in the northern or southern hemispheres, in or out of the tropics. *Just follow the sign convention, and it will all come out fine.*

For longitude, the local hour angle LHA is zero, and so, determine the sun's GHA at the instant of LAN using the almanac:
Longitude = - GHA   if GHA is less than 180
Longitude =   360 – GHA   if GHA is greater than 180

Remember, if your chronometer is inaccurate then the longitude will be off considerably since you are in essence comparing the local time with time in Greenwich. It will be off considerably anyway due to the plotting estimates.

3-Measurement Noon Sighting: Double Altitudes

There is an antiquated technique to determine latitude and longitude with a noon sighting using 3 measurements. 15 minutes before LAN, you can shoot the sun for a reference point of altitude and time. Record this as your measurement #1. Then keep track of the sun with the sextant and when it reaches the maximum altitude, record this as your measurement #2. Finally, set the sextant to the altitude setting that you had in measurement #1, and observe the sun. The moment the altitude matches the pre-positioned setting, record the time (UTC). Noon will be at the average between time measurements #1 and #3. The latitude will be derived from measurement #2.

From the almanac, determine the sun's declination DEC. For southwards meridian passage, $Sign_{noon} = +1$, and a northerly passage = -1.

Latitude = $Sign_{noon} \cdot Ho_{\#2}$ + DEC + 90 (remember the sign for DEC)
If this calculated latitude is greater than 90 degrees, then subtract 180 from it.

Time of LAN = (Time #1 + Time #2) *0.5

For longitude, the local hour angle LHA is zero, and so determine the sun's GHA at the instant of LAN using the almanac:

Longitude = - GHA   if GHA is less than 180
Longitude = 360 – GHA   if GHA is greater than 180

If for example T#1 = 19:27:31, and T#3 = 19:48:43, then the difference between them is 21 min and 12 sec. Half that is 0:10:36 difference, so add that to T#1 and you get 19:37:67 which is 19:38:07 as LAN (67 seconds = 1 min+7 sec).
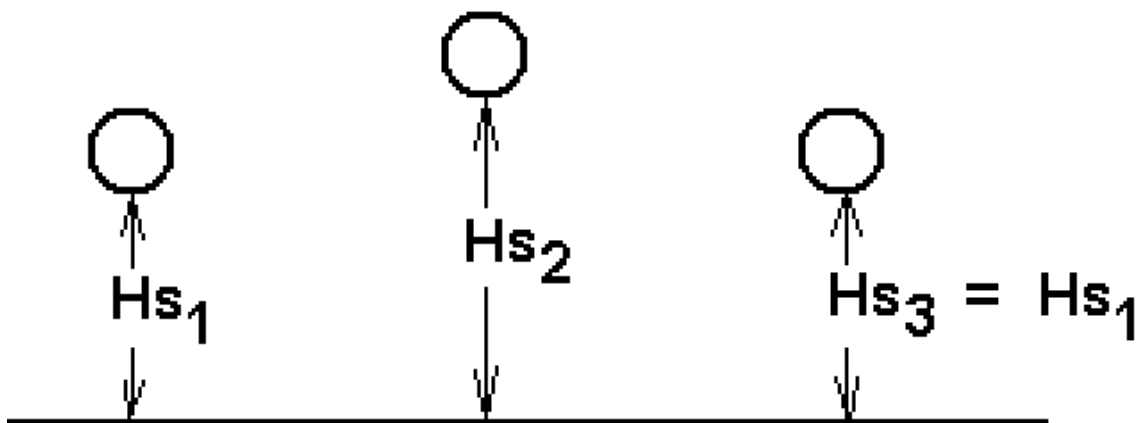
Plane Trigonometry

The simplest notion of 'trig' is the relationship of the sides and angles in a triangle. All you have to know are these three basic relationships:
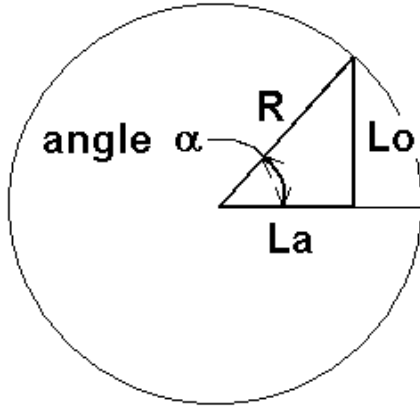
sine (α) = Lo / R   shorthand is **sin(α)**
cosine (α) = La / R   shorthand is **cos(α)**
tangent (α) = Lo / La   shorthand is **tan(α)**

The values of these trigonometric functions can be expressed as an infinite series, which your calculator will approximate by truncating the series after evaluating only a few terms. ArcSin or arcCos is the inverse function. On your calculator it might be ASIN, or InvSIN.

Useful identities:
$\sin(\alpha) = \cos(90° - \alpha)$
$\cos(\alpha) = \sin(90° - \alpha)$

Spherical Trigonometry
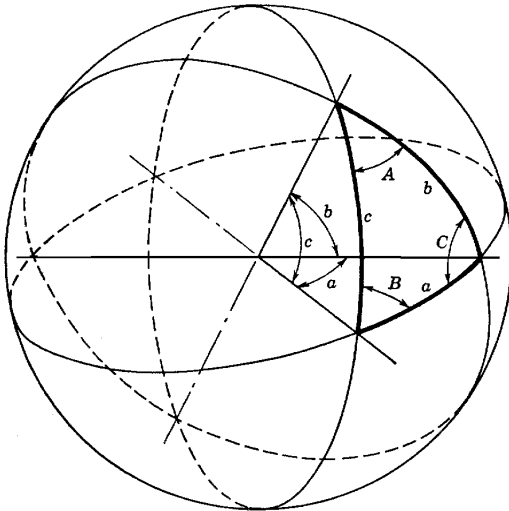
Three Great Circles on a sphere will intersect to form three inner corner angles a, b, c, and three surface angles A, B, C. Every intersecting pair of Great Circles is the same as having two intersecting planes. The angles between the intersecting planes are the same as the surface angles on the surface of the sphere. Relationships between the corner angles and surface angles have been worked out over the centuries, with the law of sines and the law of cosines (spherical trigonometry) being the most relevant to navigation.

Law of Sines: $\sin(a)/\sin(A) = \sin(b)/\sin(B) = \sin(c)/\sin(C)$

Law of Cosines: $\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(A)$

Law of Cosines in terms of co-angles by using the useful identies:
$\sin(90-a) = \sin(90-b) \cdot \sin(90-c) + \cos(90-b) \cdot \cos(90-c) \cdot \cos(A)$

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The Navigational Triangle

The navigational triangle applies spherical trigonometry, in that the inner corner angles a, b, c are related to altitude, latitude, and declination angles, and the surface angles are related to azimuth and LHA angles.
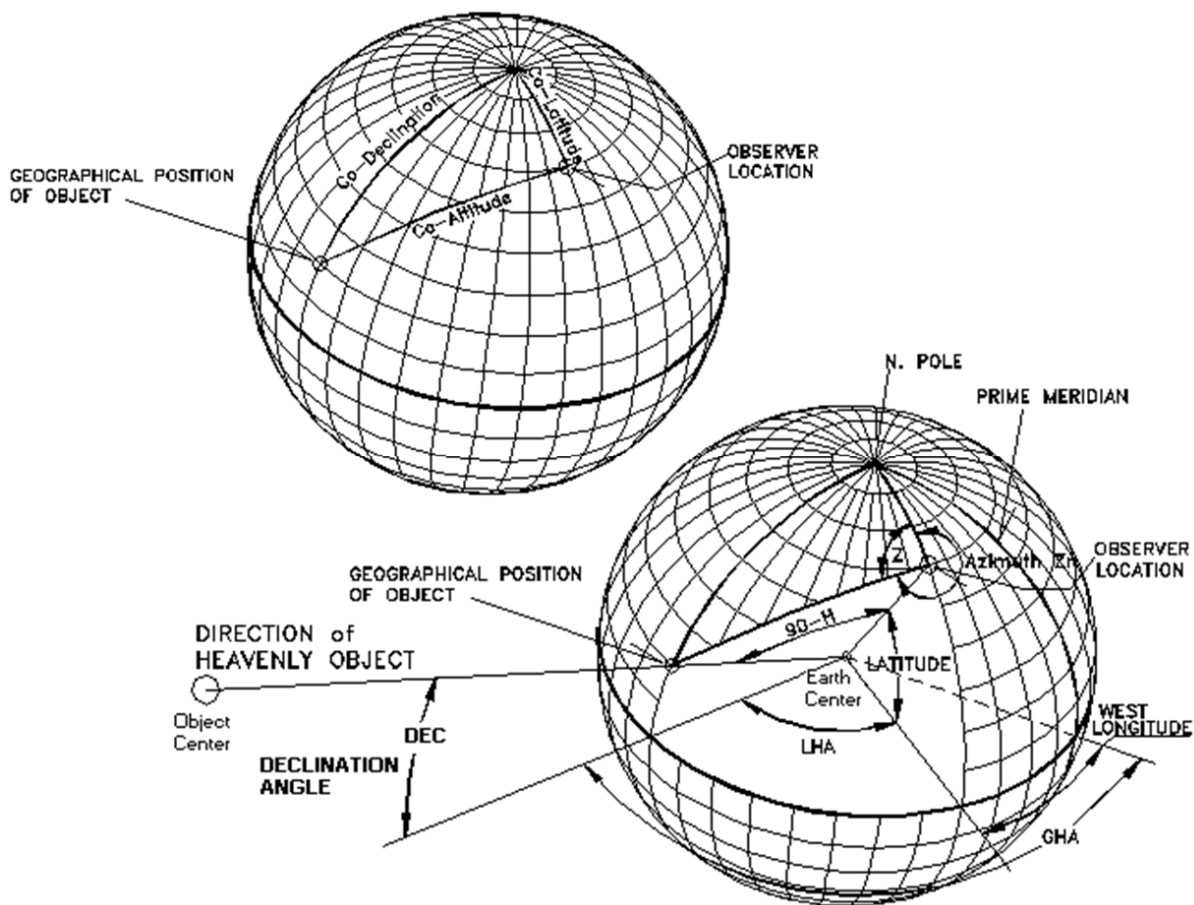
The inner corner angles corresponding to the arc sides are modifications of the altitude, latitude and declinations. As can be seen in the drawing the "co-angles" are 90° – the inner corner angle:

Co-altitude = 90° – H
Co-declination = 90° – DEC
Co-latitude = 90° - LAT

Most authorities will examine 4 cases concerning North or South declination and latitude. But if a consistent sign convention is used, we need only concern ourselves with the one picture. If you substitute a co-altitude (like 90-H) into the co-altitude formula (90-a), a = 90-H, so 90-(90-H) = H . The modified law of cosines formula for co-altitudes becomes:

$\sin(H) = \sin(Lat) \cdot \sin(DEC) + \cos(Lat) \cdot \cos(DEC) \cdot \cos(LHA)$.

Chapter Four   *Calculations for Line of Position*

   The *calculated altitude* is a way of predicting the altitude of a heavenly object by first assuming a latitude and longitude for a hypothetical observer and working out the problem backwards. The math becomes direct and unambiguous when done in this manner. The obvious choice of assumed latitude and longitude is the estimated position by dead reckoning. *Dead reckoning* is the method of advancing from a last known position by knowing the direction you headed in, how fast you were going, and how long you went. You will eventually compare this calculated altitude to a measured altitude, and so the calculated altitude must correspond to the same time as the measured altitude. This is important to extract the proper values of GHA and declination from the nautical almanac. You must be talking about the same instant in time for a correct comparison. Remembering to use the sign convention, the law of cosines gives us this relationship for the calculated altitude Hc:

$$Hc = arcSin[\ Sin(Lat_A) \cdot Sin(DEC)\ +\ Cos(Lat_A) \cdot Cos(DEC) \cdot Cos(LHA)\ ]$$

Where Lat_A is the assumed latitude, Lon_A is the assumed longitude
 and the calculated local hour angle $LHA =\ GHA + Lon_A$
If LHA is greater than 360, then subtract 360 from the calculated LHA.
DEC is of course the declination of the heavenly object.

The uncorrected azimuth angle **Zo** of a heavenly object can also be calculated as thus:

$$Zo = arcCos[\{Sin(DEC) - Sin(Lat_A) \cdot Sin(Hc)\}/\{Cos(Lat_A) \cdot Cos(Hc)\}]$$

Corrected azimuth angle **Z** (not used in any of the equations here)
If N. latitudes, then **Z = Zo** If S. latitudes, then **Z = 180 − Zo**

True Azimuth Angle from True North **Zn**
If LHA is pre-meridian passage (-, or 180<LHA < 360), **Zn = Zo**
If LHA is post-meridian passage (0<LHA < 180), **Zn = 360 − Zo**
Post meridian check can also be established if: **Sin(LHA) > 0**
Remember, arcSin or arcCos on your calculator could also be designated as ASIN, ACOS, or INVSIN, INVCOS.

By using the sign convention, we only have two cases to examine to obtain the true azimuth angle. All texts on celestial that I know of will list 4 cases due to the inconsistently applied signs on declination and latitude. Classical same name (N-N, S-S) or opposite name (N-S, S-N) rules do not apply here.

Line of Position by the Marcq Saint-Hilaire Intercept Method

This clever technique determines the true line of position from an *assumed line of position* and is the basis of modern sight reduction. Let's say you measured the altitude of the Sun at a given moment in time. You look up the GHA and declination of the Sun in the nautical almanac corresponding to the time of your altitude measurement. From an assumed position of latitude and longitude, you calculate the altitude and azimuth of the Sun according to the preceding section and arrive at **Hc** and **Zn**. On your map, you draw a line thru the pin-point assumed latitude and longitude, angled perpendicular to the azimuth angle. This is your assumed line of position. The true line of position will be offset from this line either towards the sun or away from it after comparing it to the actual *observed altitude* **Ho** (the raw sextant measurement is **Hs**, and needs all the appropriate corrections applied to make it an 'observed altitude').

The offset distance $\mathbf{D_{OFFSET}}$ to determine the true line of position is equal to:

$\mathbf{D_{OFFSET}} = 60 \cdot \mathbf{(Ho - Hc),}$ altitudes **Ho** and **Hc** in decimal degrees, or
$\mathbf{D_{OFFSET}} = \mathbf{(Ho - Hc),}$ altitudes in minutes of arc. $\mathbf{D_{OFFSET}}$ is in nautical miles for both cases.

If $\mathbf{D_{OFFSET}}$ is positive, then parallel offset your assumed line of position in the azimuth direction towards the heavenly object. If negative, then draw it away from the heavenly object. If the offset is greater than 25 nautical miles, you may want to assume a different longitude and latitude to minimize errors.

By calculating an altitude, you have created one circle of constant altitude about the geographical position, knowing that the actual circle of constant altitude is concentric to the calculated one. The difference in observed altitude and calculated altitude informs you how much smaller or larger the actual circle is. Offsetting along the radial azimuth line, the true circle will cross the azimuth line at the *intercept point*. You could also simply remember that a higher observed altitude means you are closer to the geographical position GP. If not, you are further away.

LINE OF POSITION METHODS
Intercept from Calculated Altitude

INTERCEPT DETAILS

Assumed Longitude

Assumed LOP

Assumed Latitude

DR Point

Bearing of Heavenly Object

Intercept Point

INTERCEPT LOP

Doffset (+)

Observed Circle of Constant Altitude

Calculated Circle of Constant Altitude

Near or far object, in the short distances of Doffset the light is more or less parallel

Assumed Position

Hc

Ho

Doffset (+)

(Ho-Hc)

Intercept Point
LOP perpendicular to the page at this point

Assumed Latitude

Circle of
Constant Altitude Calculated

Circle of
Constant Altitude Observed

Assumed Longitude

Azimuth direction (Bearing)
of Heavenly Object

Geographical center
of Heavenly Object

GP

Line of Position by the Sumner Line Method

If we measure the altitude of a heavenly object and make all the proper corrections, this reduces to the observed altitude **Ho**. As we should know by now, there is a circle surrounding the geographical position of the heavenly object where all observed altitudes have the same value Ho. We could practically draw the entire circle on the map, but why bother? What if instead, we draw a small arc in the vicinity of our dead reckoning position. In fact, why an arc at all, since at the map scale that interest us, a straight line will do just fine. All we need do is to rearrange the equation of calculated altitude, to make it the observed altitude instead and to solve the equation for LHA, which will give us longitude. The procedure is to input an assumed latitude, the GHA and declination for the time of observation, and out pops a longitude. Mark longitude and latitude on the map. Now input a slightly different latitude, and out pops a slightly different longitude. Mark the map, connect the dots and you have a *Sumner Line*. These are two points on the circle in the vicinity of your dead reckoning position. Or were they? Was the answer for longitude unreasonably off? Notice that for every latitude line that crosses the circle, there are 2 solutions for longitude, an east and west solution. In the arcCos function, the answer can be the angle A or the angle -A. Check both just to make sure.

East side of the circle when the object is westwards (post meridian):
$$\mathbf{Lon_C = arcCos[\{ Sin(Ho) - Sin(DEC) \cdot Sin(Lat_A)\}/\{Cos(Lat_A) \cdot Cos(DEC)\}] - GHA}$$

West side of the circle when the object is eastwards (pre meridian):
$$\mathbf{Lon_C = -arcCos[\{ Sin(Ho) - Sin(DEC) \cdot Sin(Lat_A)\}/\{Cos(Lat_A) \cdot Cos(DEC)\}] - GHA}$$

Where $\mathbf{Lat_A}$ is the assumed latitude, $\mathbf{Lon_C}$ is the calculated longitude
 $\mathbf{DEC}$ is of course the declination of the heavenly object.

The two values for assumed latitude could be the dead reckoning latitude $\mathbf{Lat_{DR}}$ + 0.1 and – 0.1 degree.

The advantage to this method is that the LOP comes out directly without offsets. There is no azimuth calculation, just two calculations with the same equation having slightly differing latitude arguments. Also, the fact that only the assumed latitude is required means no estimated position of the longitude is needed at all. This method turns into an E-W LOP when near the meridian passage, just like a noon shot.

LINE OF POSITION METHODS
SUMNER LINE METHOD

SUMNER LINE DETAILS

Longitude #1

Latitude Argument #1

Observed Circle of Constant Altitude

Assumed Latitude

SUMNER LOP

Latitude Argument#2

Longitude #2

Calculated Longitude #1

Assumed Latitude

Latitude trial #1

Calculated Longitude #2

Latitude trial #2

Circle of Constant Altitude Observed

Geographical center of Heavenly Object

GP
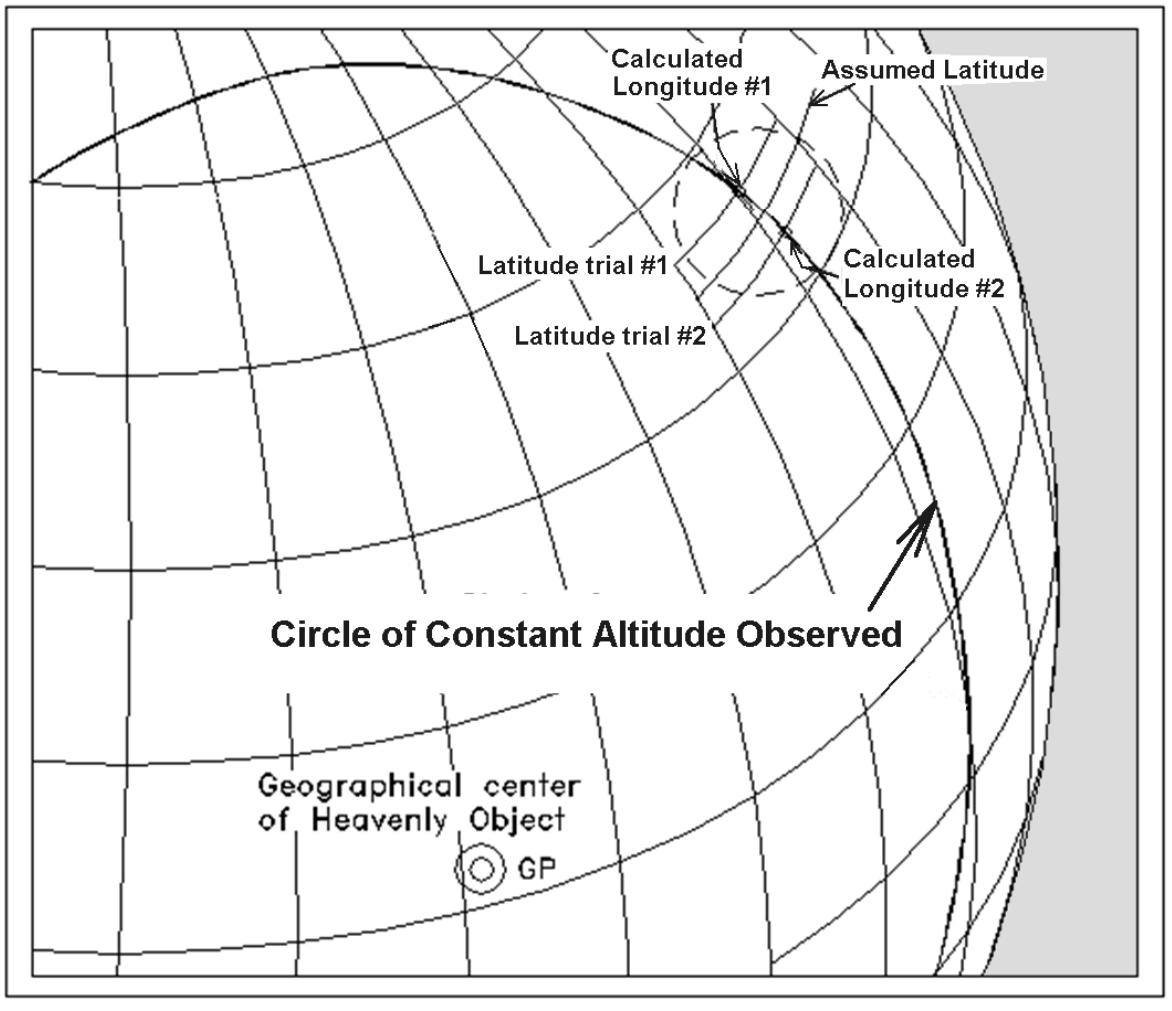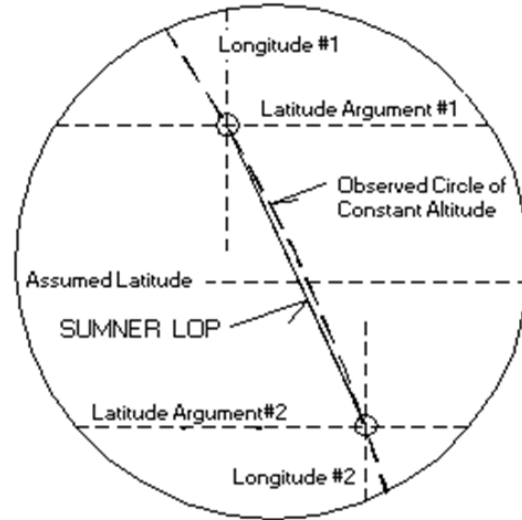
History of the Sumner Line

The Sumner line of position takes its name from Capt. Thomas H. Sumner, an American ship-master, who discovered the technique serendipitously and published it. I recount here his discovery, paraphrased from his book:

Capt. Sumner sailed from Charleston S.C. on November 25th, 1837, and was bound for Greenock, Ireland. After passing 21 deg west longitude, he had no observations due to thick weather until he came close to land. He was within 40 miles of the Tuskar lighthouse off the coast of Ireland by dead reckoning with the weather getting worse at around midnight December 17th. At that point the wind backed from the south to the south east making the coast a lee shore. He kept close to the wind tacking back and forth until daylight, and then kept on a course of ENE. At about 10 am local time he was able to make a sun shot observation, but going so long since the last observation, he was unsure of his dead reckoning latitude. A longitude (Lon1) with his uncertain latitude (Lat1) was calculated:

$$Lon_1 = -arcCos[\{ Sin(Ho) - Sin(DEC) \cdot Sin(Lat_1)\}/\{Cos(Lat_1) \cdot Cos(DEC)\}] - GHA$$

Declination and GHA of the sun was from the almanac and the time mark from the sun shot. The longitude was 15' east of his dead reckoning position. He then assumed a second latitude (Lat2) 10' north of his dead reckoning towards the coast:

$$Lon_2 = -arcCos[\{ Sin(Ho) - Sin(DEC) \cdot Sin(Lat_2)\}/\{Cos(Lat_2) \cdot Cos(DEC)\}] - GHA$$

Marking the chart with the location Lat1, Lon1, and then with Lat2, Lon2, he noticed the 2nd position was 27 miles ENE of the 1st position

He did this a 3rd time with another 10' more northerly latitude assumption and calculated a 3rd longitude:

$$Lon_3 = -arcCos[\{ Sin(Ho) - Sin(DEC) \cdot Sin(Lat_3)\}/\{Cos(Lat_3) \cdot Cos(DEC)\}] - GHA$$

After plotting this third point on the chart (Lat3, Lon3), he noticed that all three of the points were on a line. This line just happened to cross Smalls Light as well. Capt. Sumner rightfully concluded that all three points saw the same observed altitude of the Sun, and so where he might not know exactly where he was, he knew he was somewhere on that line. Coincidentally his course was on that line as well, and he continued to sail ENE; within an hour he saw Smalls Light and made his landfall. Thus the Sumner Line method was discovered accidently by practice.

Chapter 5 *Measuring Altitude with the Sextant*

The sextant is a wonderfully clever precision optical instrument for which we can thank Sir Issac Newton for the design. It reflects the image of the Sun (or anything, really) twice with two flat mirrors in order to combine it with a straight-thru view, allowing you to see the horizon and heavenly object simultaneously in the same pupil image. This allows for a 'shake-free' view, as the horizon and Sun move together in the combined image. The straight-thru view is accomplished with the second mirror *(horizon mirror)*, which is really a half mirror, silvered on the right and clear on the left. You see the horizon unchanged on the left, and the twice-reflected sun on the right if you use a *'traditional'* mirror as opposed to a *'whole horizon'* mirror. With a whole horizon mirror, both horizon and Sun will be in the entire view. It does this by partial silvering of the entire horizon mirror like some sunglasses are, reflecting some light and transmitting the rest. This makes the easy shots easier, but the more difficult shots with poor illumination or star shots more difficult. Even with the traditional mirror, curiously, you will see a whole image of the sun in the pupil that you can move to the right or left by rocking the sextant side to side. The glass surface itself is reflective. When it is at its lowest point, you are correctly holding the sextant and can take a reading. The horizon however, will only be on the left side of the image. In order to determine the altitude of the Sun, you change the angle of the first mirror *(index mirror)* with the *index arm* until the Sun is close to the horizon in the pupil image. Now turn the precision index drum (knob) until the lower *limb* of the Sun just kisses the horizon. Rock it back and forth to make sure you have the lowest reading. In order not to burn your eye out (that would be stupid…), there are *filters* (shades) that can be rotated over the image path of the index mirror. Likewise, there are other filters that cover the horizon mirror to remove the glare and increase the contrast between horizon and sky.

SUN SHOT, VIEW THRU THE SEXTANT EYE PIECE

The Nautical Sextant

low magnification telescope

combined view

Index mirror

Sky filters

Horizon mirror

Horizon

Horizon filters

Handle

arc

degree scale

Index arm

Squeeze to disengage micrometer thread

Vernier scale

Index drum

micrometer adjustment of arcminute scale
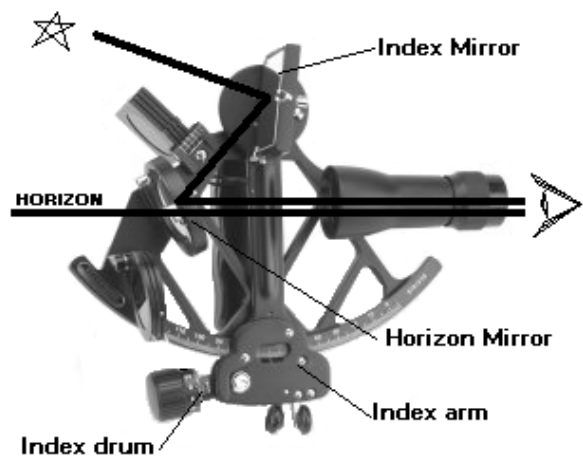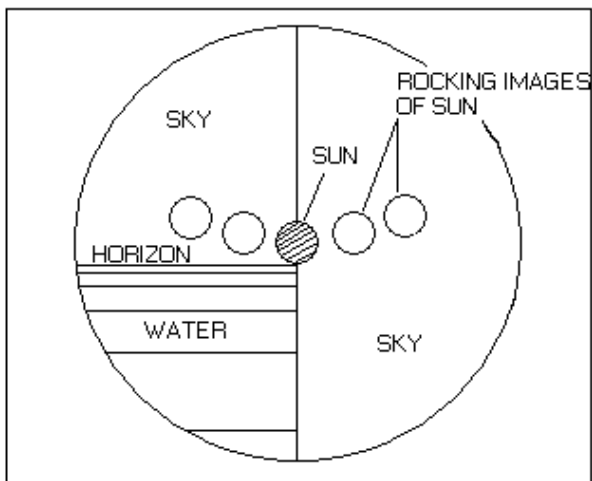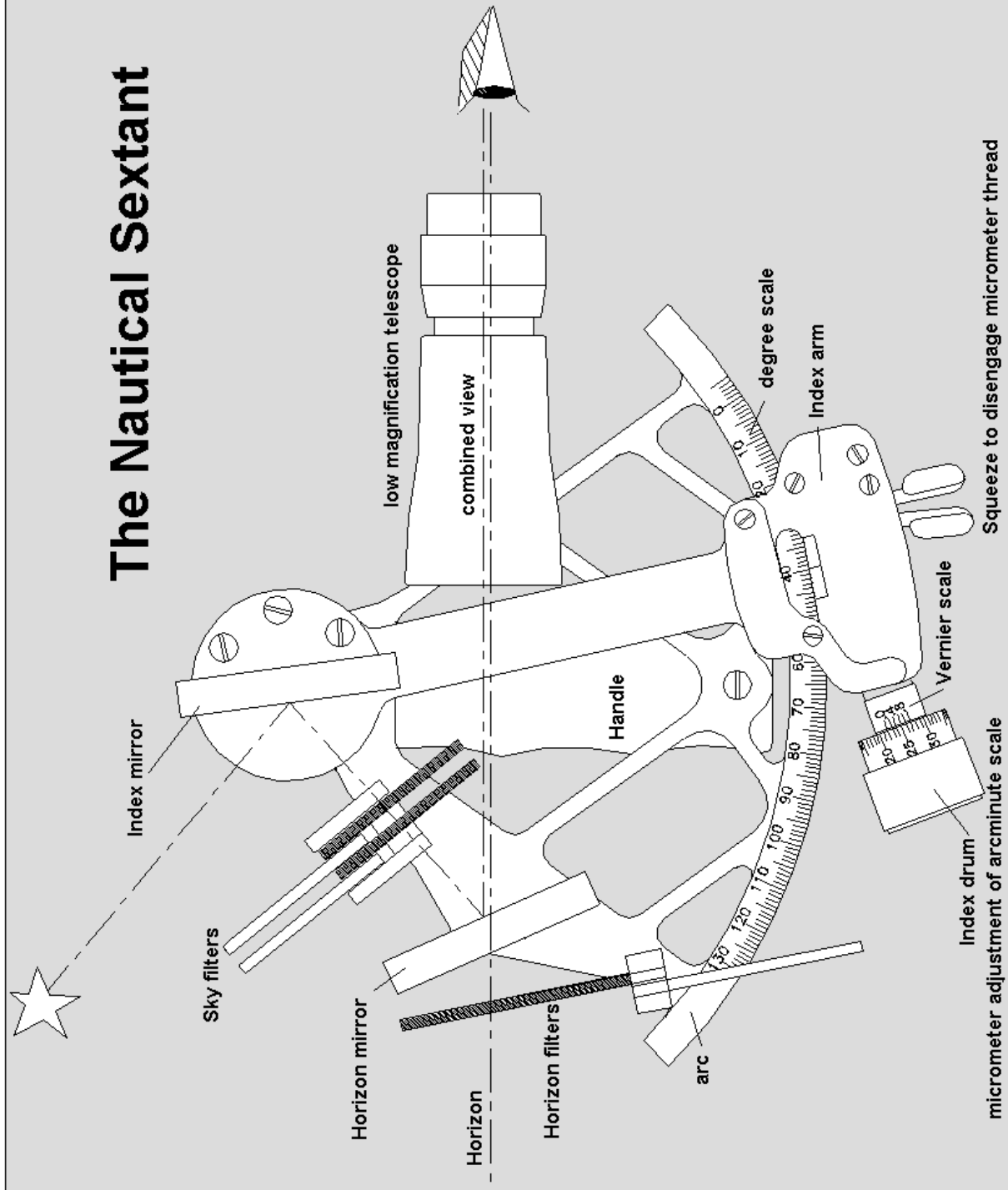
Mirror Alignments

Even an expensive precision instrument will give you large errors (although consistent systematic error) unless it is adjusted and calibrated. Before any round of measurements are taken, you should get into the habit of calibrating and if necessary adjusting the mirrors to minimize the errors.

The first check is to see if the index mirror is perpendicular to the sextant's arc. Known as *Perpendicularity Alignment*, it is checked in a round-about manner by finding the image of the arc in the index mirror when viewed externally at a low angle. Set the arc to about 45 degrees. The reflected arc in the index mirror should be in line with the actual arc. This can be tricky, as it only works if the mirrored surface is exactly along the pivot axis of the index arm. Since most mirrors are *secondary surface* mirrors (the silvering is on the back of the glass), you need to compare the position of the rear of the glass to the pivot axis first to see if this technique will work. *First surface* mirrors (the silvering is on the front of the glass) seem to be an upgrade, but the sextant's manufacturer may not have necessarily redesigned the mirror-holding mount. This positions the index mirror reflecting surface 2 to 3 mm or so in front of the pivot axis. In that case, the reflected image of the arc should be slightly below the viewed actual arc. There are precision-machined cylinders about an inch high that you can place on the arc and view their reflections. The reflections should be parallel to the actual cylinders. If not, then turn the set screw behind the index mirror to bring it into perpendicular alignment.

The next alignment is *Side Error Alignment* of the horizon mirror. This can be done two ways after setting the arc to the zero angle point such that you see the same object on the left and right in the pupil image. First, at sea in the daytime, point the sextant at the horizon. You will see the horizon on the left and the reflected horizon on the right. Adjust the index drum until they are in perfect alignment while holding the sextant upright. Now roll (tilt) the sextant side to side. Is the horizon and reflected image still line-to-line? If not, then side error exists. This is corrected with adjustments to the set screw that is perpendicularly away from the plane of the arc on the horizon mirror. Second method is to wait until nighttime, where a point source that is nearly infinitely far away presents itself (yes, I mean a star). Same procedure as before except that you need not roll the sextant. What you will see is two points of light. The horizontal separation is the *side error*, and the vertical separation is the *index error*. Adjust the drum knob to negate the index error effect until the star and its reflection are vertically line-to-line but still separated horizontally. Make adjustments to the side-error set screw until the points of light converge to a single image point.

HORIZON

WATER

SEXTANT SET AT ZERO, SHOWING INDEX ERROR
VIEWING THE HORIZON UPRIGHT

SEXTANT WITH INDEX ERROR REMOVED
VIEWING HORIZON TILTED AT ANGLE TO
REVEAL SIDE ERROR
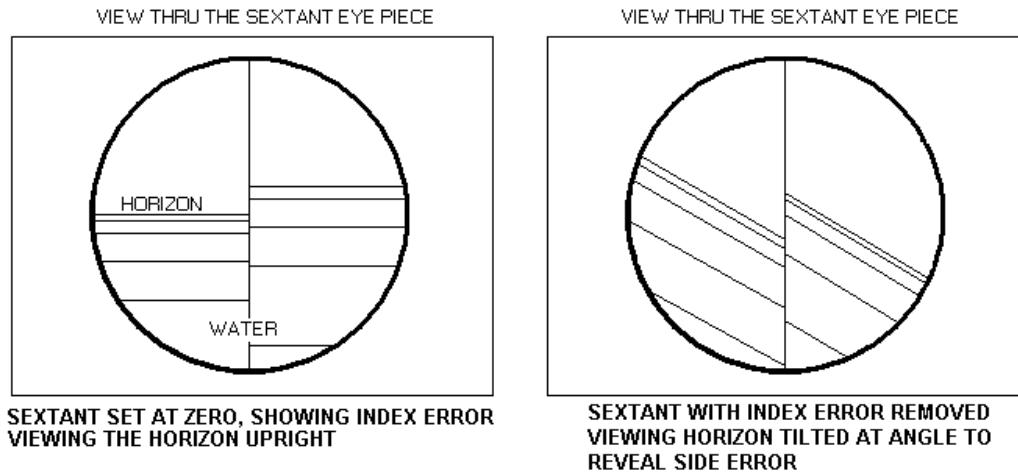
You could stop here at this point, reading the drum to determine the index error IE (Note: index correction IC = - IE). Or you could continue to zero out the index error as well with a last series of adjustments. In which case, for the *Index Error Alignment*, set the arc to zero (index arm and drum to the zero angle position). You will notice that the star image now has two points separated vertically. Adjusting the remaining set screw on the horizon mirror (which is near the top of the mirror), you can eliminate the vertical separation. Unfortunately this last set screw does not only change the vertical separation, but it slightly affects the horizontal separation as well. Now you need to play around with both set screws until you zero-in the two images simultaneously. With a little practice these procedures will be easy and routine. A word of caution: the little wrench used to adjust the set screws maybe very difficult to replace if you should drop it overboard. Making a little hand lanyard for the wrench will preserve it. Maybe…

Note: I have also used high altitude jet aircraft, their contrails, and even cloud edges to adjust the mirrors (low accuracy…). If you have dark enough horizon shades, you can even use the sun's disk to adjust the mirrors.

SIDE ERROR

INDEX ERROR

SEXTANT SET AT ZERO
VIEWING A STAR
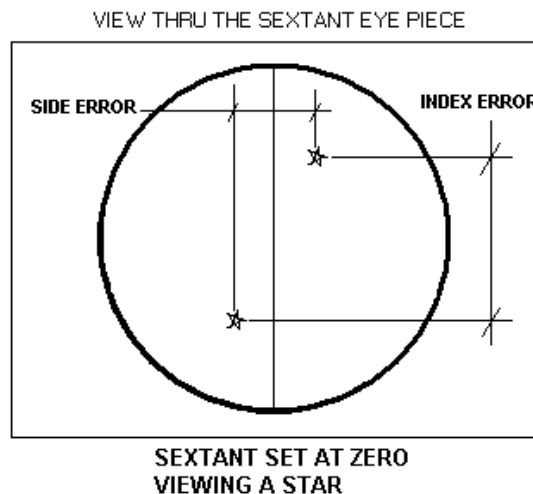
Sighting Techniques

Bringing the object down
Finding the horizon is much easier than finding the correct heavenly object in the finder scope. So, the best technique is to first set the index arm to zero degrees and sight the object by pointing straight at it. Then keeping it in view, 'lower' it down to the horizon by increasing the angle on the index arm until the horizon is in sight. Careful with the sun, as you don't want to see it unfiltered thru the horizon glass; keep the sun on the right hand side of the mirror using the darkest shade over the index mirror.

Rocking for the lowest position
Rocking the sextant from side to side will help you determine when the sextant is being pointed in the right direction and held proper, as the object will find its lowest point. This will give the true sextant altitude Hs.

Letting her rise, letting her set
Often it is easier to set the sextant 'ahead' of where the heavenly object is going, and to simply let her rise or set as the case may be to the horizon. At that point you mark the time. That way you can be rocking the sextant to get the true angle without also fiddling with the index drum. This leaves a hand free, sort of, to hold the chronometer such that at the time of mark, you just have to glance to the side a little to see the time.

Upper limb, lower limb
With an object such as the Sun or Moon, you can choose which limb to use, the lower limb or upper limb. Unless the Sun is partially obscured by clouds, the lower limb is generally used. Depending on the phase of the moon, either lower limb or upper limb is used.
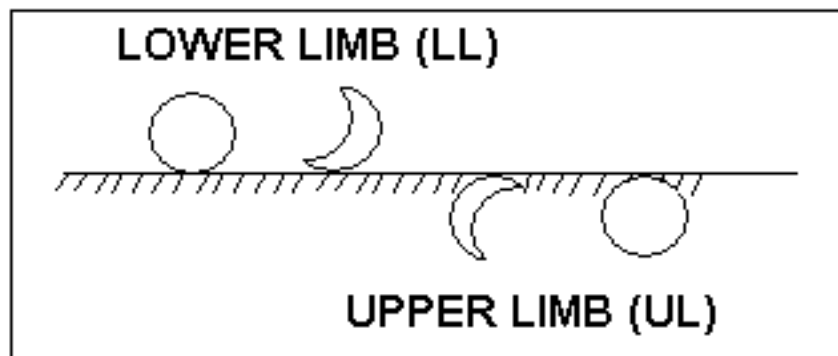
Brief History of Marine Navigational Instruments

The earliest instrument was the *astrolabe*, constructed in the Middle East during the 9th century AD. It was a mechanical rotating slide rule with a pointer to determine the altitude of stars against a protractor. Contemporary was a very simple instrument, the *quadrant*. It was a quarter of a circle protractor with a plumb-bob and a pair of peep sights to line up with Polaris. The first real ancestor to the modern sextant was the *cross staff*, described in 1342. A perpendicular sliding cross piece over a straight frame allowed one to line up two objects and determine the angle. Of course one had to look at both objects simultaneously by dithering the eyeball back and forth – a bit of a problem. Also one had to look into the blinding sun. Since a cross staff looked like a crossbow, one was said to be 'shooting the sun', an expression still used today. The *Davis backstaff* in 1594 was an ingenious device where sun shots were taken with your back to the sun, using the sun's shadow over a vane to cast a sharp edge (so the navigator wouldn't go blind!). The navigator would line up the horizon opposite the sun azimuth with a pair of peep holes, and rotated a shadow vane on an arc until the shadow edge lined up on the forward peep hole. This limited one to only sun shots to determine latitude. In the 1600's a French soldier-mathematician by the name of Vernier invented the *vernier scale*, whereby one could easily interpolate between degree scales to a 1/10 or 1/20 between the engraved lines on the protractor scale. The search for determining longitude created bizarre proposals, but it was recognized that determining the time was the answer, and so one needed an <u>accurate</u> clock. A clock could be mechanical, or astronomical. The Moon is about ½ degree of arc across its face, and moves across the celestial sphere at the rate of about one lunar diameter every hour (~0.5 arcminute per minute of time). Therefore its arc distance to another star could be used as a sort of astronomical clock. Tables to do this were first published in 1764. The calculations and corrections are indeed frightening, and this method of determining time to within several minutes of Greenwich Mean Time is called doing *Lunars*, and those who practice it are *Lunarians*. Undoubtedly if you used this method too often you would have been branded a *Lunatic*. Fortunately in 1735 John Harrison invented the first *marine chronometer*, having some wood elements and weighing 125 lbs. He worked on it for 40 years (until he produced the alarm-clock size H4)! The *Hadley Octant* in 1731 was the first to use the double reflecting principle as described by Isaac Newton a century before. It could measure across 90 degrees of arc, even though it was only physically 45 degrees arc, an 1/8 of a circle. The sextant with it's ability to record angles of 120 degrees came about for use in doing lunars, and so was a contemporary of the octant. By 1780, refinements such as tangential screws, vernier scales, and shades glasses, fixed the design of sextants and octants for the next 150 years.
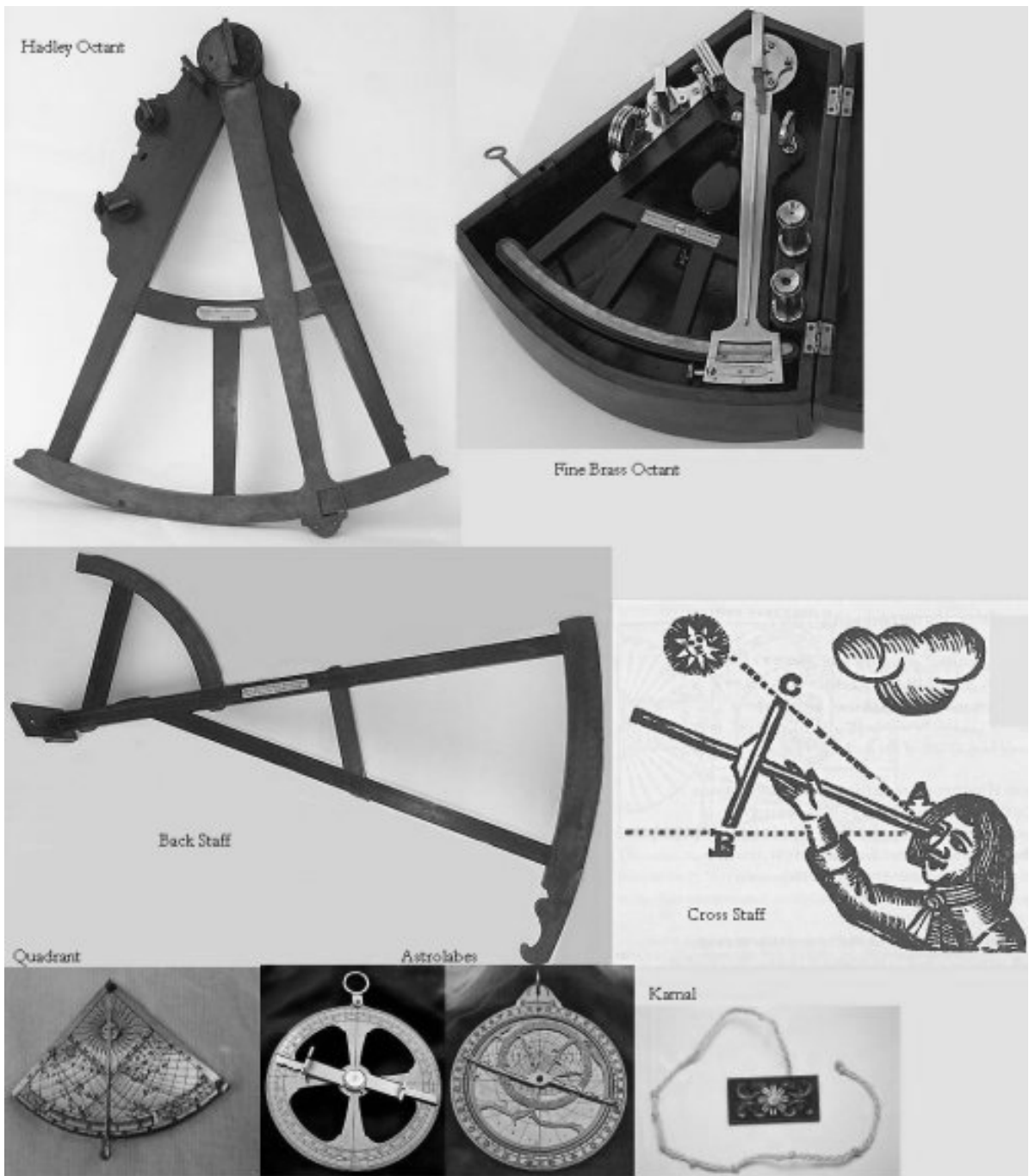
Octants, Back Staff, Cross Staff, Quadrant, Astrolabes, Kamal

Chapter 6   *Corrections to Measurements*

There are numerous corrections to be made with the as-measured altitude **Hs** that you read off of the sextant's arc degree scale and arc minute drum and vernier. Your zero point on the scale could be off, the same as the bathroom scale when you notice that it says you weigh 3 lbs even before you get on it. This is known as index error IE, and the correction is **IC** (IC = -IE). For our example of the bathroom scale, **IC** = -3. The other major corrections are **parallax, semi-diameter, refraction**, and **dip**, listed from the largest effect to the smallest. Lunar parallax can be at most a degree, semi-diameter ¼ degree, refraction and dip are on the order of 1/20th degree.
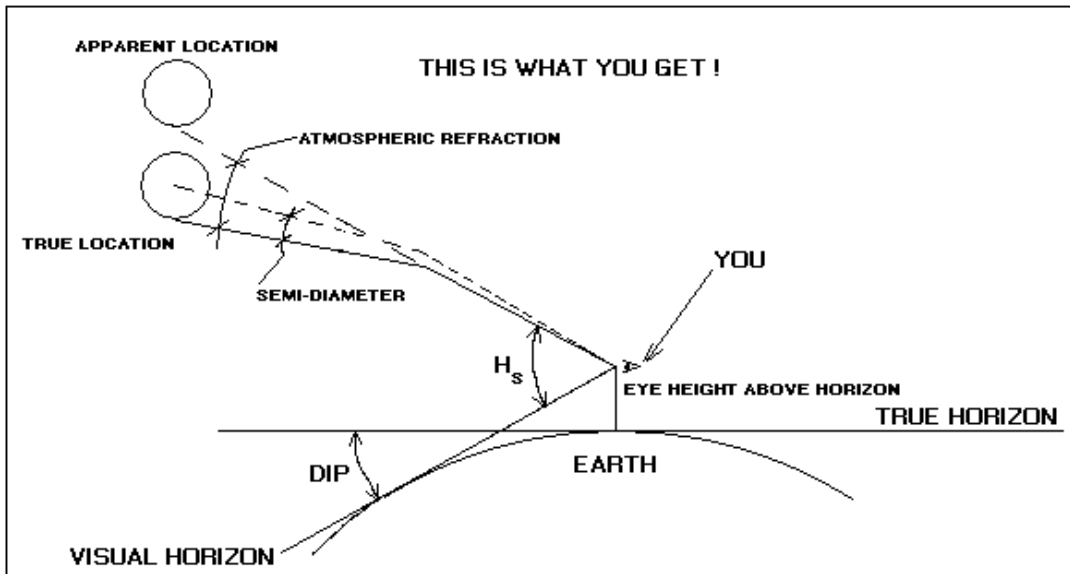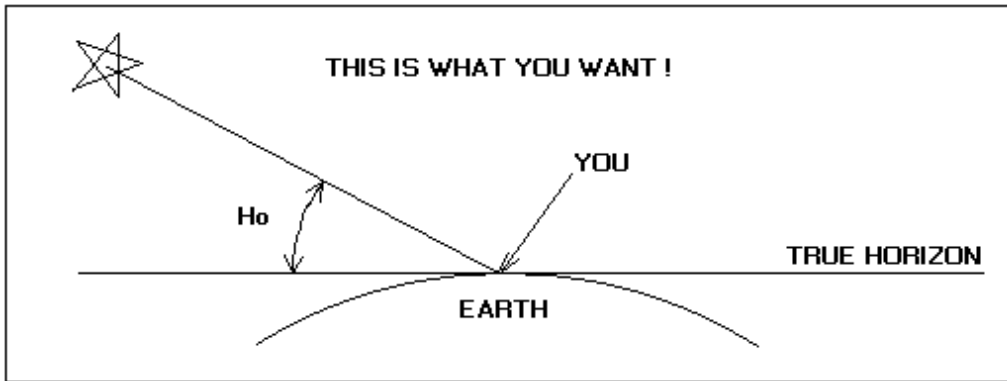
Note, the **Hs** in the figure conveniently does not account for the index error.

The sextant basically has an index correction **IC** and an instrument correction **I**. The instrument error is due to manufacturing inaccuracies and distortions, and should be listed on a calibration sheet from the manufacturer. Generally it's negligible. Index error is due to the angular misalignment of the index mirror, with respect to the zero point on the scale. The correction IC is negative when the zero is "on the scale", and positive when "off the scale". By adjusting the drum knob as described on pages 40 and 41 to negate the optical index error, one can see if the zero is on or off the scale.

Dip Correction

Dip is the angle of the visual horizon, dipping below the true horizon due to your eye height above it. This is also tabulated in the nautical almanac. An approximate equation for dip correction that incorporates a standard horizon refraction is thus:

Corr ^{DIP} = - 0.0293 · SquareRoot(h_{eye})   in Decimal Degrees
Corr ^{DIP} = - 1.758 · SquareRoot(h_{eye})   in arc minutes
Where **h_{eye}** is the eye height above the water, meters.
Corr ^{DIP} is always negative.

Distance to visible horizon as a function of eye height above the water:

$$d_{horiz} = 2.118 \cdot \sqrt{h_{eye}}$$

Distance to horizon d_{horiz}, n.mi
eye height h_{eye}, meters

Altitude Corrections

Let us first define the apparent altitude, **Ha = Hs + IC + Corr ^{DIP}**
Ha is the altitude <u>without</u> corrections for refraction, semi-diameter, or parallax. The atmosphere bends (refracts) light in a predictable way. These corrections are tabulated on the 1st page of the nautical almanac based on the apparent altitude Ha. The corrections vary for different seasons, and whether you are using the lower or upper *limb* of the Sun for your observations. Since measurements are made to the edge (limb) and not the center of the Sun, the angle of the Sun's visual radius (semi-diameter) must be accounted for. The table also lists slight deviations from the nominal for listed planets. There are special lunar correction tables at the end of the almanac, which include the effects of lunar semi-diameter, parallax and refraction. The variable name for all of these combined altitude error corrections, lunar, solar or otherwise, is **Corr ^{ALT}**, sometimes called the 'Main Correction'.
The true observed altitude is a matter of adding up all the corrections:
 Ho = Ha + Corr ^{ALT}

Tables of Altitude and Dip Correction, averaged values

For simplified corrections, use these tables instead of the Nautical Almanac.

Altitude correction for sun and stars
Corr^{ALT}

Ha, deg.	Sun LL	Sun UL	Stars
10	+11'	-21'	-5'
13	+12'	-20'	-4'
15	+12.5'	-19.5'	-3.5'
17	+13'	-19'	-3'
20	+13.5'	-18.5'	-2.5'
24	+14'	-18'	-2'
31	+14.5'	-17.5'	-1.5'
41	+15'	-17'	-1'
59	+15.5'	-16.5'	-0.5'
85	+16'	-16'	0

Dip Correction

Height	Corr^{DIP}
0.7m	-1.5'
1.3m	-2'
2.0m	-2.5'
2.9m	-3'
3.9m	-3.5'
5.1m	-4'
6.4m	-4.5'

$$Ha = Hs + IC + Corr^{DIP}$$
$$Ho = Ha + Corr^{ALT}$$

For the moon use the almanac which will include SD, R, and PA

Graph of Dip Correction, for when land is used as horizon

Dip Correction, arcmin
with land or sea horizon

$$Dip = 60 \cdot atan\left(\frac{h_{eye}}{1852 \cdot d_h} + \frac{d_h}{8268}\right)$$

eye height h$_{eye}$, meters

distance to land/horizon d$_h$, n.mi
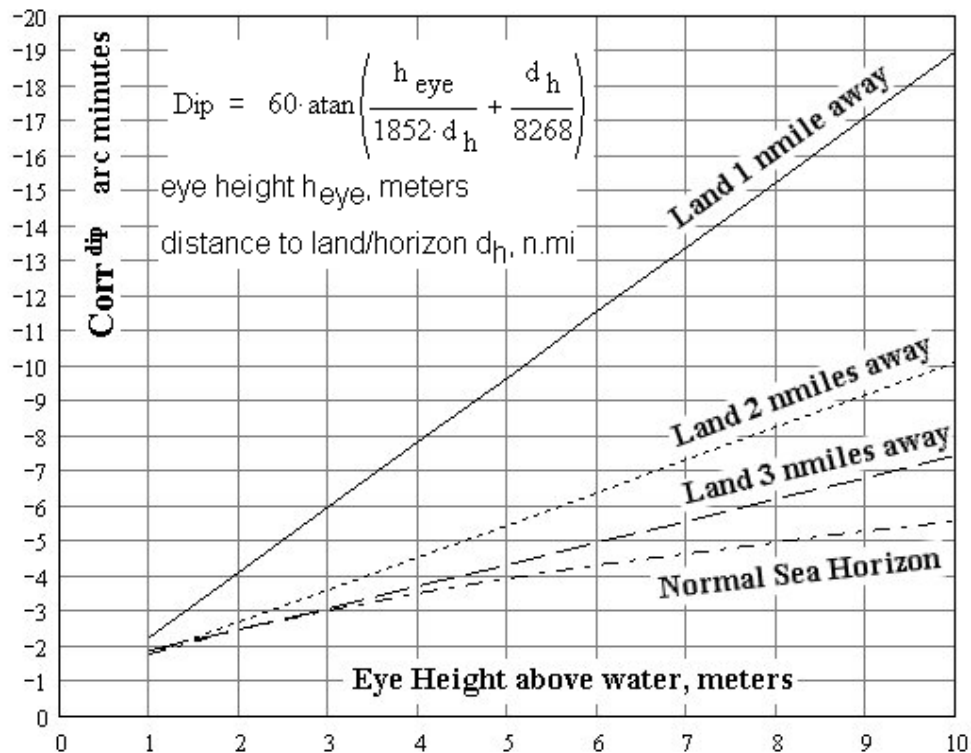
Refinements
Corrections for observations can be calculated instead of using tables, and refinements can be employed for non-standard conditions.

Start with the apparent altitude Ha:
Ha = Hs +IC + Corr ^{DIP}   (assume instrument correction I ~ 0)

The *horizontal parallax* for the Moon is given in the nautical almanac tables as the variable **HP** in minutes of arc, and you must convert it to decimal degrees. HP for the Sun = 0.0024 degrees, but this is rarely included as being so small a value. For Venus, the HP is hidden in the altitude correction tables, listed as 'Additional Corr^n'. Use the largest number at zero altitude to = HP_{Venus}. To determine the *parallax-in-altitude* **PA**, use this equation:

PA = HP · Cos(Ha) · (1 −(Sin^2(Lat))/298.25) includes earth oblateness

The semi-diameter of the Sun **SD** is given at the bottom of the page of the tables in the nautical almanac in minutes of arc, and you must convert it to decimal degrees. So is the semi-diameter daily average of the Moon, but you can calculate one based on the hourly value of **HP**:
The semi-diameter of the Moon: **SD = 0.2724 · HP · (1 + Sin(Ha)/60.5)**
The terms in the parenthesis are "augmentation", meaning the observer is a very little closer to the moon with greater altitude angle. This is a small term.

Atmospheric refraction is standardized to surface conditions of 10 deg C and 1010mb pressure. This standard refraction correction **Ro** is thus:
Ro = - 0.0167 / Tan[Ha + 7.31/(Ha+4.4)] degrees

The correction for non-standard atmospheric conditions is referred to as **f**:
f = 0.28 · Pressure$_{mb}$ / (Temperature$_{DEG\ C}$ + 273)
The final refraction correction **R** is thus:
R = Ro · f   This number is always negative.

If the lower limb were observed, then **sign$_{limb}$ = +1**
If the upper limb were observed, then **sign$_{limb}$ = -1**

Observed altitude with refinements:
Ho = Ha + R + PA + SD · sign$_{limb}$

Here we see that the altitude correction **Corr ^{ALT} = R + PA + SD · sign$_{limb}$**
Note: Convert arcminutes to decimal degrees for consistent calculations.

Artificial horizon

A fun way of practicing sighting the Sun while on land is to use an artificial horizon. This is simply a pan of water or old motor oil that you place down on the ground in view of the Sun. Since the liquid will be perfectly parallel with the true horizon (no dip corrections here), it can be used as a reflecting plane. In essence you point the sextant to the pan of liquid where you see the reflection of the Sun. Move the index arm until you bring the real Sun into the pupil image with the index mirror. With the micrometer drum bring both images together (no semi-diameter corrections either) and take your reading. This gives a reading twice the apparent altitude. Undoubtedly you will need to position extra filters over the horizon mirror to darken the Sun's image, as normally you would be looking at a horizon. Correct the reading by taking the apparent altitude Ha and divide by two, then add the refraction correction:

Ha = (Hs + IC)/2   no dip correction
Ho = Ha + R   no semi-diameter correction

The wind is very bothersome, as it will ripple the water's surface and therefore the reflected image. Protective wind guards around the pan work somewhat, but generally you may have to wait minutes for a perfect calm. What works best is **mineral oil in a protected pan set up on a tripod** so that you can get right up to it. The ripples dampen out almost immediately.

To be very accurate, you can let the sun touch limb-to-limb. If pre-meridian (morning) then let the bottom image rise onto the reflected image, measure the time, and SUBTRACT a semi-diameter (UL): **Ho = Ha + R - SD**
If post meridian (afternoon), let the top image set onto the reflected image, measure the time, and ADD a semi-diameter (LL): **Ho = Ha + R + SD**

ARTIFICIAL HORIZON AND SEXTANT

2x Ha

PAN OF WATER OR MOTOR OIL OR MINERAL OIL

Reflection

Sun

UL
MORNING

Sun

Reflection

LL
AFTERNOON

Chapter 7   *Reading the Nautical Almanac*
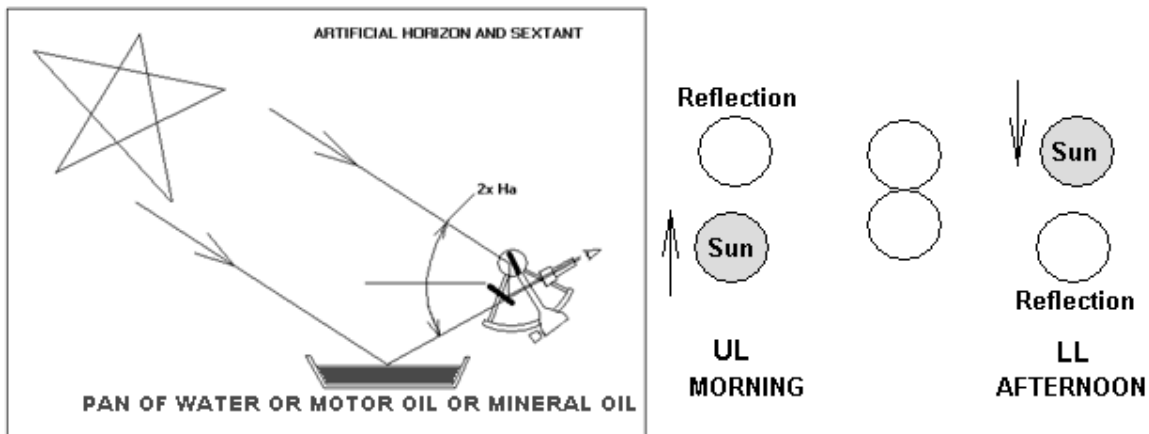
The nautical almanac has detailed explanations in the back regarding how to read the tabular data and how to use the interpolation tables (increments and corrections). The data is tabulated for each hour on the dot for every day of the year, and you must interpolate for the minutes and seconds between hours. Every left hand page in the almanac is similar to all other left hand pages, and the same for all right hand pages. Three days of data are presented for every left and right hand page pairs. The left page contains tabular data of GHA and declination for Aries (declination = 0), Venus, Mars, Jupiter, Saturn and 57 selected stars. The right page has similar data for the Sun and Moon. It also provides the **Local Mean Time** (LMT) for the events of sunrise, sunset, moonrise, and moonset at the prime meridian. For your particular locality, you can express the event time in UT with the following equation:

EventTime$_{UT}$ = LMT – Longitude/15.   Hours UT at your longitude. Remember the sign convention, West -, East +.

Interpolation tables, *v* and *d* corrections
Probably the most confusing part of the tables is *interpolation* for times between hourly-tabulated data, and how to properly apply the mysterious *v* and *d* corrections. The interpolation tables ('increments and corrections') are based on nominal rates of change of GHA for the motions of the Sun and planets, Moon, and Aries. This way, only one set of interpolation tables is required, with variances to the rates compensated with the *v* and *d* values. These are hourly variances, and their applicable fraction (the correction **Corr** ^{V} and **Corr** ^{d}) is given in the interpolation tables for the minute of the hour. The *v* number refers to *v*ariances in the nominal GHA rate. There is no nominal rate for changes in declination, so *d* is the direct hourly rate of change of declination. For GHA, the interpolation tables will tabulate increments (**Corr** ^{GHA}) down to the second of each minute. The *v* and *d* correction is interpolated only for every minute. Take the hourly data in the tables, GHA, add the interpolated increment for the minutes and seconds, and finally add the interpolated *v* correction. Similarly for declination, take the tabulated hourly value Dec and add the interpolated *d* correction. Our sign convention imposes that a south declination is negative, and a north declination is positive. A word of caution, the value of *d* (with our sign convention) may be positive or negative. If the tabulated hourly data for declination is advancing northwards (less southwards), then the sign is positive. We could have a negative declination (south), but have a positive *d* if declination is becoming less southwards. Along the same line, we could have a positive declination (north) but a negative *d* if the declination is

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heading south (less northwards). Unfortunately the almanac has no sign for d so you must devine the correct sign by looking at the progression of DEC.

The final values at the particular hour, minute, and second are thus:
GHA = GHA$_{hour}$ + Corr ^{GHA} + Corr ^{V} (For stars **GHA = SHA + GHA^{Aries}**)
DEC = DEC$_{hour}$ + Corr ^{d}
Where GHA$_{hour}$ and DEC$_{hour}$ are the table values in the almanac for the hour.

After all the interpolations and corrections are performed, convert the angles to decimal degrees and make sure the sign convention was applied consistently to the declination value.

Note: In the nautical almanac, liberal use is made of the correction factor **Corr^{n}**. It seems to appear everywhere and applied to everything. The n is actually a variable name for any of the parameters that require 'correction'. Notably, **Corr ^{DIP}**, **Corr ^{ALT}**, **Corr ^{GHA}**, **Corr ^{V}**, and **Corr ^{d}**.

Since we like to use our calculators, instead of using the 'increments and corrections' table (it's actually very easy) we can interpolate for ourselves in the following manner. Let's say we shot an observation at Universal Time **H** hours, **M** minutes, and **S** seconds (H:M:S). The nautical almanac tables for the particular day gave us a GHA in degrees and arcminutes at the UT hour. We convert it to decimal degrees and call it **GHA$_{hour}$**. We do the same for the declination and call it **DEC$_{hour}$**. Note the hourly variance v and declination rate d in arcminutes per hour. We can also define the hour fraction, **Δt**, which are the minutes and seconds in decimal form: **Δt = (M/60) + (S/3600)**. Now, the correct interpolated value for our specific time of observation is thus:
GHA = GHA$_{hour}$ + {Rate + (v/60)} · Δt   decimal degrees

Where   **Rate** = 15.00000   (degrees/hour) for Sun or planets
          Rate = 14.31667   (degrees/hour) for Moon
          Rate = 15.04107   (degrees/hour) sidereal rate for Aries

In a similar line, declination is interpolated thus:
DEC = DEC$_{hour}$ + (d/60) · Δt   (DEC$_{hour}$ and d with the proper sign)

Note, $v/60$ and $d/60$ converts arcminutes per hour to degrees per hour. Carry out all calculations to 4 decimal places, and make sure the sign convention was applied correctly (carpenter's rule: measure twice, cut once).
Visit an on-line Nautical Almanac at: http://www.tecepe.com.br/scripts/AlmanacPagesISAPI.isa

Left Page   2000 JUNE 8, 9, 10 (THURS., FRI., SAT.)

Annotation labels overlaid on the page: DEGREES, ARCMINUTES, DAY, UT HOUR, GHA variance per hour, DECLINATION change per hour, Brightness Magnitude, North Declination, South Declination, LMT of meridian passage.

UT	ARIES GHA	VENUS −3.9 GHA	Dec	MARS +1.6 GHA	Dec	JUPITER −2.0 GHA	Dec	SATURN +0.2 GHA	Dec	STARS Name	SHA	Dec
8 THURSDAY												
00	256 40.9	181 15.5	N22 43.6	172 58.3	N24 01.4	203 42.5	N18 13.5	204 32.3	N16 50.2	Acamar	315 26.4	S40 18.2
01	271 43.4	196 14.7	44.0	187 58.9	01.5	218 44.4	13.6	219 34.4	50.3	Achernar	335 34.7	S57 14.0
02	286 45.8	2..			01.6	233 46.3	13.7	234 36.6	50.3	Acrux	173 20.5	S63 06.3
03	301 48.3	2..			01.7	248 48.2	13.9	249 ..			255 20.9	S28 58.5
04	316 50.8	2..			01.8	263 50.1	14.0	264 ..			291 01.4	N16 30.5
05	331 53.2	256 11.3	45.8	248 01.3	01.9	278 52.0	14.2	279 ..				
06	346 55.7	271 10.5	N22 46.0	263 02.0	N24 02.0	293 53.8	N18 14.3	294 ..			166 29.3	N55 57.8
07	1 58.2	286 09.7	46.4	278 02.6	02.0	308 55.7	14.4	309 ..			153 06.6	N49 19.0
08	17 00.6	301 08.8					14.6	324 ..		Al Na'ir	27 56.3	S46 57.4
09	32 03.1	316 08.0					14.7	339 51.7	50.8	Alnilam	275 57.0	S 1 12.2
10	47 05.5	331 07.1					14.8	354 53.8	50.9	Alphard	218 06.2	S 8 39.7
11	62 08.0	346 06.3	48.0	338 05.0	02.4	9 03.2	15.0	9 56.0	51.0			
12	77 10.5	1 05.4	N22 48.4	353 05.6	N24 02.5	24 05.1	N18 15.1	24 58.1	N16 51.1	Alphecca	126 19.3	N26 43.0
13	92 12.9	16 04.6	48.8	8 06.3	02.6	39 07.0	15.2	40 00.3	51.1	Alpheratz	357 54.1	N29 05.3
14	107 15.4	31 03.8	49.2	23 06.9	02.7	54 08.9	15.4	55 02.4	51.2	Altair	62 17.9	N 8 52.1
15	122 17.9	46 02.9	49.6	38 07.5	02.7	69 10.8	15.5	70 04.6	51.3	Ankaa	353 25.9	S42 18.1
16	137 20.3	61 02.1	49.9	53 08.1	02.8	84 12.7	15.7	85 06.7	51.4	Antares	112 38.4	S26 25.9
17	152 22.8	76 01.2	50.3	68 08.7	02.9	99 14.5	15.8	100 08.9	51.4			
18	167 25.3	91 00.4	N22 50.7	83 09.3	N24 03.0	114 16.4	N18 15.9	115 11.1	N16 51.5	Arcturus	146 04.8	N19 11.0
19	182 27.7	105 59.6	51.1	98 10.0	03.1	129 18.3	16.1	130 ..			207 48.8	S69 01.7
20	197 30.2	120 58.8	51.5	113 10.6	03.2	144 20.2	16.2	145 ..			234 22.6	S59 30.9
21	212 32.6	135 58.0	51.9	128 11.2	03.3	159 22.1	16.3	160 ..			278 43.3	N 6 20.9
22	227 35.1	150 57.0	52.2	143 11.8	03.3	174 24.0	16.5	175 ..			271 12.7	N 7 24.3
23	242 37.6	165 56.2	52.6	158 12.4	03.4	189 25.8	16.6	190 ..				
9 FRIDAY												
00	257 40.0	180 55.3	N22 53.0	173 13.0	N24 03.5	204 27.7	N18 16.7	205 24.0	N16 51.9	Canopus	264 01.2	S52 41.9
01	272 42.5	195 ..			03.6	219 29.6	16.9	220 26.2	52.0	Capella	280 50.0	N45 59.8
02	287 45.0	210 ..			03.7	234 31.5	17.0	235 28.3	52.1	Deneb	49 38.1	N45 16.7
03	302 47.4	225 ..			03.8	249 33.4	17.2	250 30.5	52.2	Denebola	182 44.0	N14 34.3
04	317 49.9	240 51.9	54.5	233 15.5	03.8	264 35.3	17.3	265 32.6	52.2	Diphda	349 06.2	S17 59.1
05	332 52.4	255 51.1	54.9	248 16.1	03.9	279 37.1	17.4	280 34.8	52.2			
06	347 54.8	270 50.2	N22 55.3	263 16.7	N24 04.0	294 39.0	N18 17.6	295 36.9	N16 52.4	Dubhe	194 04.2	N61 45.3
07	2 57.3	285 49.4	55.6	278 17.3	04.1	309 40.9	17.7	310 39.1	52.4	Elnath	278 25.9	N28 36.4
08	17 59.8	300 48.5	56.0	293 17.9	04.2	324 42.8	17.8	325 41.3	52.4		90 50.3	N51 29.4
09	33 02.2	315 47.7	56.4	308 18.6	04.3	339 44.7	18.0	340 43.4	52.5		33 57.0	N 9 52.5
10	48 04.7	330 46.8	56.7	323 19.2	04.3	354 46.6	18.1	355 45.6	52.5		15 35.1	S29 37.1
11	63 07.1	345 46.0	57.1	338 19.8	04.4	9 48.4	18.2	10 47.8	52.6			
12	78 09.6	0 45.1	N22 57.5	353 20.4	N24 04.5	24 50.3	N18 18.4	25 49.9	N16 52.6	Gacrux	172 12.1	S57 07.1
13	93 12.1	15 44.3	57.8	8 21.0	04.6	39 52.2	18.5	40 52.0	52.9	Gienah	176 02.7	S17 32.7
14	108 14.5	30 43.5	58.2	23 21.6	04.6	54 54.1	18.6	55 54.2	52.9	Hadar	149 02.0	S60 22.6
15	123 17.0	45 42.6	58.6	38 22.3	04.7	69 56.0	18.8	70 56.3	53.0	Hamal	328 12.5	N23 27.6
16	138 19.5	60 41.7	58.9	53 22.9	04.8	84 57.9	18.9	85 58.5	53.1	Kaus Aust.	83 56.9	S34 23.0
17	153 21.9	75 40.9	59.3	68 23.5	04.9	99 59.7	19.1	101 00.7	53.2			
18	168 24.4	90 40.0	N22 59.6	83 24.1	N24 05.0	115 01.6	N18 19.2	116 02.8	N16 53.2	Kochab	137 18.6	N74 09.5
19	183 26.9	105 39.2	23 00.0	98 24.7	05.0	130 03.5	19.3	131 05.0	53.3	Markab	13 48.5	N15 12.3
20	198 29.3	120 38.3	00.4	113 25.3	05.1	145 05.4	19.5	146 07.1	53.4	Menkar	314 26.0	N 4 05.3
21	213 31.8	135 37.5	00.7	128 25.9	05.2	160 07.3	19.6	161 09.3	53.4	Menkent	148 19.3	S36 22.4
22	228 34.3	150 36.6	01.1	143 26.6	05.3	175 09.2	19.7	176 11.4	53.5	Miaplacidus	221 42.3	S69 43.4
23	243 36.7	165 35.8	01.4	158 27.2	05.3	190 11.1	19.9	191 13.6	53.6			
10 SATURDAY												
00	258 39.2	180 34.9	N23 01.8	173 27.8	N24 05.4	205 12.9	N18 20.0	206 15.8	N16 53.6	Mirfak	308 55.4	N49 51.5
01	273 41.6	195 34.1	02.1	188 28.4	05.5	220 14.8	20.1	221 17.9	53.7	Nunki	76 10.6	S26 17.7
02	288 44.1	210 33.2	02.5	203 29.0	05.6	235 16.7	20.3	236 20.1	53.8	Peacock	53 34.8	S56 43.8
03	303 46.6	225 32.4	02.8	218 29.6	05.6	250 18.6	20.4	251 22.2	53.9	Pollux	243 40.5	N28 01.6
04	318 49.0	240 31.5	03.2	233 30.3	05.7	265 20.5	20.5	266 24.4	53.9	Procyon	245 10.7	N 5 13.4
05	333 51.5	255 30.7	03.5	248 30.9	05.8	280 22.4	20.7	281 26.6	54.0			
06	348 54.0	270 29.8	N23 03.9	263 31.5	N24 05.9	295 24.2	N18 20.8	3..				33.6
07	3 56.4	285 29.0	04.2	278 32.1	05.9	310 26.1	20.9	3..				12.2
08	18 58.9	300 28.1	04.6	293 32.7	06.0	325 28.0	21.1	3..				50.3
09	34 01.4	315 27.3	04.9	308 33.3	06.1	340 29.9	21.2	3..				43.5
10					06.1	355 31.8	21.3					
11					06.2	10 33.7	21.5					
12					N24 06.3	25 35.6	N18 21.6	26 41.7	N16 54.5	Schedar	349 52.4	N56 32.0
13					06.4	40 37.4	21.7	41 43.8	54.6	Shaula	96 35.3	S37 06.2
14					06.4	55 39.3	21.9	56 46.0	54.7	Sirius	258 33.0	S16 43.1
15	124 16.1	45 22.1	06.9	38 37.0	06.5	70 41.2	22.0	71 48.1	54.7	Spica	158 34.8	S11 09.8
16	139 18.6	60 21.3	07.3	53 37.7					54.8	Suhail	223 00.2	S43 26.3
17	154 21.1	75 20.4	07.6	68 38.3					54.9			
18	169 23.5	90 19.5	N23 07.9	83 38.9			N18		N16 54.9	Vega	80 45.4	N38 47.0
19	184 26.0	105 18.7	08.2	98 39.5					55.0	Zuben'ubi	137 16.4	S16 02.6
20	199 28.5	120 17.8	08.6	113 40.1					55.1		SHA	Mer.Pass.
21	214 30.9	135 17.0	08.9	128 40.7	06.9	160 52.5	22.8	162 01.1	55.2	Venus	283 15.3	11 57
22	229 33.4	150 16.1	09.2	143 41.4	07.0	175 54.4	23.0	177 03.2	55.2	Mars	275 33.0	12 27
23	244 35.9	165 15.3	09.6	158 42.0	07.0	190 56.3	23.1	192 05.4	55.3	Jupiter	306 47.7	10 21
Mer.Pass.	6h 48.2m	v −0.8	d 0.4	v 0.6	d 0.1	v 1.9	d 0.1	v 2.2	d 0.1	Saturn	307 44.0	10 17

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2000 JUNE 8, 9, 10 (THURS., FRI., SAT.)

SUN and MOON

UT	SUN GHA	SUN Dec	MOON GHA	v	MOON Dec	d	HP
d h	° ′	° ′	° ′	′	° ′	′	′
8 00	180 15.0	N22 51.1	100 37.4	9.4	N13 23.9	10.4	58.8
01	195 14.9	51.3	115 05.8	9.4	13 13.5	10.4	58.7
02	210 14.8	51.5	129 34.2	9.6	13 03.1	10.4	58.7
03	225 14.7 ..	51.8	144 02.8	9.6	12 52.7	10.6	58.7
04	240 14.5	52.0	158 31.4	9.7	12 42.1	10.5	58.6
05	255 14.4	52.2	173 00.1	9.8	12 31.6	10.7	58.6
06	270 14.3	N22 52.4	187 28.9	9.9	N12 20.9	10.7	58.6
07	285 14.2	52.7	201 57.8	10.0	12 10.2	10.7	58.5
08	300 14.1	52.9	216 26.8	10.1	11 59.5	10.8	58.5
09	315 13.9 ..	53.1	230 55.9	10.1	11 48.7	10.9	58.5
10	330 13.8	53.3	245 25.0	10.2	11 37.8	10.9	58.4
11	345 13.7	53.5	259 54.2	10.3	11 26.9	10.9	58.4
12	0 13.6	N22 53.7	274 23.5	10.4	N11 16.0	11.1	58.4
13	15 13.5	53.9	288 52.9	10.5	11 04.9	11.0	58.3
14	30 13.4	54.1	303 22.4	10.5	10 53.9	11.1	58.3
15	45 13.2 ..	54.4	317 51.9	10.6	10 42.8	11.1	58.3
16	60 13.1	54.6	332 21.5	10.7	10 31.7	11.2	58.2
17	75 13.0	54.8	346 51.2	10.8	10 20.5	11.3	58.2
18					11.2	58.2
19					11.3	58.1
20					11.4	58.1
21					11.3	58.1
22					11.5	58.0
23					11.4	58.0
9 00					11.5	58.0
01	195 12.0	56.5	102 51.6	11.3	8 49.6	11.5	57.9
02	210 11.9	56.7	117 21.9	11.5	8 38.1	11.6	57.9
03	225 11.8 ..	56.9	131 52.4	11.5	8 26.5	11.5	57.9
04	240 11.6	57.1	146 22.9	11.5	8 15.0	11.7	57.8
05	255 11.5	57.3	160 53.4	11.7	8 03.3	11.6	57.8
06	270 11.4	N22 57.5	175 24.1	11.7	N 7 51.7	11.7	57.8
07	285 11.3	57.7	189 54.8	11.7	7 40.0	11.6	57.7
08	300 11.1	57.9	204 25.5	11.9	7 28.4	11.8	57.7
09	315 11.0 ..	58.1	218 56.4	11.9	7 16.6	11.7	57.7
10	330 10.9	58.3	233 27.3	11.9	7 04.9	11.7	57.6
11	345 10.8	58.5	247 58.2	12.0	6 53.2	11.8	57.6
12	0 10.6	N22 58.7	262 29.2	12.1	N 6 41.4	11.8	57.6
13	15 10.5	58.9	277 00.3	12.2	6 29.6	11.8	57.5
14	30 10.4	59.1	291 31.5	12.2	6 17.8	11.9	57.5
15	45 10.3 ..	59.3	306 02.7	12.2	6 05.9	11.8	57.5
16	60 10.1	59.5	320 33.9	12.3	5 54.1	11.9	57.4
17	75 10.0	59.7	335 05.2	12.4	5 42.2	11.9	57.4
18	90 09.9	N22 59.9	349 36.6	12.4	N 5 30.3	11.9	57.4
19	105 09.8	23 00.1	4 08.0	12.5	5 18.4	11.9	57.3
20	120 09.7	00.3	18 39.5	12.5	5 06.5	11.9	57.3
21	135 09.5 ..	00.4	33 11.0	12.6	4 54.6	12.0	57.3
22	150 09.4	00.6	47 42.6	12.6	4 42.6	11.9	57.2
23	165 09.3	00.8	62 14.2	12.7	4 30.7	12.0	57.2
10 00	180 09.2	N23 01.0	76 45.9	12.7	N 4 18.7	11.9	57.2
01	195 09.0	01.2	91 17.6	12.8	4 06.8	12.0	57.2
02	210 08.9	01.4	105 49.4	12.8	3 54.8	12.0	57.1
03	225 08.8 ..	01.6	120 21.2	12.9	3 42.8	11.9	57.1
04	240 08.7	01.8	134 53.1	12.9	3 30.9	12.0	57.1
05	255 08.5	02.0	149 25.0	12.9	3 18.9	12.0	57.0
06	270 08.4	N23 02.1	163 56.9	13.0	N 3 06.9	12.0	57.0
07	285 08.3	02.3	178 28.9	13.1	2 54.9	12.0	57.0
08					2 42.9	12.0	56.9
09					2 30.9	12.0	56.9
10					2 18.9	12.0	56.9
11					2 06.9	11.9	56.9
12					1 55.0	12.0	56.8
13					1 43.0	12.0	56.8
14	30 07.4	03.6	230 14.0	13.3	1 31.0	12.0	56.8
15			294 46.3				
16			309 18.7				
17			323 51.1				
18			338 23.5				
19	105 06.8	04.5	352 55.9				
20	120 06.6	04.7	7 28.4				
21	135 06.5 ..	04.8	22 00.9				
22	150 06.4	05.0	36 33.4	13.5	S 0 04.6	11.9	56.5
23	165 06.3	05.2	51 05.9	13.6	S 0 16.5	11.9	56.5
SD	15.8	d 0.2	SD 15.9		15.7		15.5

Twilight / Sunrise / Moonrise

Lat.	Naut.	Civil	Sunrise	8	9	10	11
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	08 58	11 03	12 57	14 48
N 70					11 13	13 00	14 43
68					11 21	13 02	14 39
66					11 28	13 03	14 35
64					11 34	13 05	14 32
62					11 39	13 06	14 30
60					11 43	13 07	14 28
N 58	////	01 47	02 59	10 24	11 47	13 08	14 26
56	////	02 15	03 15	10 30	11 50	13 08	14 24
54	00 59	02 36	03 29	10 35	11 53	13 09	14 22
52	01 38	02 53	03 41	10 40	11 56	13 10	14 21
50	02 04	03 07	03 51	10 45	11 59	13 10	14 20
45	02 47	03 36	04 13	10 54	12 04	13 12	14 17
N 40	03 17	03 58	04 31	11 02	12 09	13 13	14 15
35	03 40	04 16	04 46	11 09	12 12	13 14	14 13
30	03 55	04 31	04 58	11 15	12 16	13 14	14 11
20	04 25	04 55	05 20	11 25	12 22	13 16	14 08
N 10	04 48	05 15	05 38	11 34	12 27	13 17	14 06
0	05 07	05 33	05 56	11 42	12 32	13 18	14 03
S 10	05 24	05 50	06 13			13 19	14 01
20	05 39	06 07	06 31			13 21	13 59
30						13 22	13 56
35					12 51	13 23	13 54
40				12 16	12 54	13 24	13 52
45				12 30	12	13	48
S 50				12 39	13 06	13 27	13 47
52				12 43	13 09	13 28	13 46
56	06 51	07 40	08 27	12 53	13 11		
58	06 58	07 50	08 41	12 59	13 15		
S 60	07 05	08 01	08 58	13 05	13 19		

Sunset / Twilight / Moonset

Lat.	Sunset	Civil	Naut.	8	9	10	11
°	h m	h m	h m	h m	h m	h m	h m
N 72	□	□	□	02 21	02 00	01 44	01 29
N 70	□	□	□	01 58	01 47	01 39	01 30
68	□	□	□	01 40	01 37	01 34	01 31
66	23 32	////	////	01 25	01 29	01 31	01 32
64	22 20	////	////	01 13	01 22	01 28	01 33
62	21 45	////	////	01 03	01 15	01 25	01 33
60	21 20	22 57	////	00 54	01 10	01 23	01 34
N 58	21 00	22 13	////	00 47	01 05	01 20	01 34
56	20 44	21 45	////	00 40	01 01	01 19	01 35
54	20 30	21 24	23 02	00 34	00 57	01 17	01 35
52	20 18	21 06	22 22	00 28	00 53	01 15	01 36
50	20 08	20 52	21 56	00 23	00 50	01 14	01 36
45	19 46						01 37
N 40	19 28						01 37
35	19 11						01 38
30	19 07						01 38
20	18 33						01 39
N 10	18 22						01 40
0	17 40						01 40
S 10	17 40						01 41
20	17 28	17 52	18 19	23 52	24 48	00 48	01 41
30	17 07	17 33	18 03	23 44	24 44	00 44	01 42
35	16 55	17 23	17 54	23 40	24 42	00 42	01 42
40	16 41	17 11	17 45	23 34	24 39	00 39	01 43
45	16 25	16 58	17 35	23 28	24 37	00 37	01 43
S 50	16 04	16 42	17 22			00 33	01 44
52	15 55	16 33	17 16			00 31	01 44
54	15 44					00 30	01 44
56	15 32					00 28	01 45
58	15 17					00 26	01 45
S 60	15 01	15 57	16 57	22 58	24 23	00 23	01 45

SUN / MOON

	SUN Eqn. of Time 00h	12h	Mer. Pass.	MOON Mer. Pass. Upper	Lower	Age	Phase
d	m s	m s	h m	h m	h m	d	%
8	01 00	00 55	11 59	17 54	05 29	06	43
9	00 49	00 43	11 59	18 43	06 19	07	54
10	00 37	00 31	11 59	19 29	07 06	08	64

Annotations:
- Lunar Horizontal parallax
- Lunar variance
- Lunar declination change, negative (less northerly)
- day
- Solar events good for a 3 day period
- hour
- minute
- Events of sunset and moonset at the particular latitude
- Solar declination hourly change
- LMT of noon on the 8th
- Solar semi diameter
- Lunar semi-diameter for 3 days

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Chapter 8   *Sight Reduction*

The process of taking the raw observational data and turning the information into a Line Of Position (LOP) is called *sight reduction*. Even though the equations and methods have been described all throughout the book, what is needed here most is <u>organization</u> to minimize the calculation random errors.

History

Trigonometric tables were first published by Regiomontanus in the mid 1400's, followed by the early logarithm tables of Edmund Gunter in the late 1600's, which allowed multiplication to be treated as addition problems. This is the basis of the slide rule (does anybody remember those??). French almanacs were published in the late 1600's where the original zero longitude 'rose line' ran thru Paris. The English almanacs were published later in the 1700's. The altitude-difference method of determining a line of position introduced the age of improved navigation, described in 1875 by Commander Adolphe-Laurent-Anatole Marcq de Blonde de Saint-Hilaire, of the French Navy. This 'Marcq Saint-Hilaire' method remains the basis of almost all celestial navigation used today. But the Sumner line method may be considered equally easy, 2 computations for the Saint-Hilaire method, and 2 for the Sumner line method. Computed altitude and azimuth angle have been calculated by means of the log sine, cosine, and haversine (½ [1-cos(a)] also called natural haversine) tables.

Sight reduction was greatly simplified early in the 1900's by the coming of the various short-method tables - such as the Weems Line of Position Book, Dreisonstok's *Hydrographic Office* method H.O. 208 (1928), and Ageton's H.O. 211 (1931). Almost all calculations were eliminated when the inspection tables, H.O. 214 (1936), H.O. 229, and H.O. 249 were published, which tabulated zillions of pre-computed solutions to the navigational triangle for all combinations where LHA and latitude are whole numbers. The last two methods, H.O. 229 and H.O. 249 developed in the mid 1940's and early 1950's remain the principle tabular method used today. The simplest tabular method of all is to use a shorthand version of Ageton's tables known as the S-tables, which are only 9 pages long. No whole number assumptions are required, and the answers are the same as a navigational calculator. You must do some minor addition, though, and the tables are a bit of a maze (takes practice).

The following page is an example of a sight-reduction form using the "calculator method" instead of the typical HO 229, 249 tabular methods.

SIGHT REDUCTION BY CALCULATOR, INTERCEPT METHOD

Sun / Moon / Planet / Star   LL / UL   UT Date _____m _____d_____yr
Time of observation   UTC = _____h _____m _____s
DR position [1]   Lat = _____   Lon = _____

Eye height H_{eye} _____meters
Index correction   IC = _____ arcmin (remember + or – sign)
Sextant measured altitude Hs = _____deg _____arcmin

Dip correction from the corrections table (always -): $Corr^{DIP}$ = - _____
Apparent altitude Ha =   Hs + IC + $Corr^{DIP}$ **Ha**   = _____
Altitude correction from the corrections table: $Corr^{ALT}$ = _____
True altitude Ho = Ha + $Corr^{ALT}$ **Ho**   = _____

From the almanac tabular data, at the h hour on the UT date:
GHA table = _____ v = _____
DEC table = _____ [1] d = _____ (careful of the sign)
SHA = _____ if star
Increment of GHA for the m minutes and s seconds $Corr^{GHA}$ = _____
Additional increment due to variation v $Corr^{v}$ =   _____
[2] GHA = GHA table + $Corr^{GHA}$ + $Corr^{v}$ **GHA** =   _____

Increment of DEC for m minutes due to rate d is $Corr^{d}$ = _____
DEC = DEC table + $Corr^{d}$ **DEC** =   _____
_____

Local Hour angle LHA = GHA + Lon   **LHA** = _____

                (repeat Ho here to subtract Hc from)   **Ho**_____

arcSin[Sin(DEC) · Sin(Lat) + Cos(Lat) · Cos(DEC) · Cos(LHA)]   = Hc_____

Ho – Hc = _____ **· 60 =** Offset Distance   **= Doffset**_____n.miles

arcCos[{Sin(DEC) – Sin(Lat) · Sin(Hc)}/{Cos(Lat) · Cos(Hc)}]   = Zo_____

True Azimuth Angle from True North **Zn**
If LHA is pre-meridian passage (-, or 180<LHA < 360), **Zn = Zo**
If LHA is post-meridian passage (0<LHA < 180), **Zn = 360 – Zo**   **Zn =** _____

Notes: [1] North is+, South is -. East is +, West is -
            [2] GHA table = SHA + GHA Aries   for a star

Sun Shot Example

Let's say this is the data:
DR position Lat = 44.025° N,   Lon = -67.850° W
Eye height = 2 meters   Greenwich date 7/15/2001
Index correction IC = +3.4' (off the arc)
Time of observation UTC = 14h 15m 37s
Sextant measured altitude of the sun Hs = 52° 52.3' Lower Limb

Altitude corrections from the abridged corrections table:
$Corr^{DIP}$ = -2.5' $Corr^{ALT}$ = +15.3' (interpolate in your head)
Observed true altitude Ho = 52° 52.3' + 3.4' -2.5' +15.3' = 53° 8.5' = 53.1416°
(Remember adding a negative number is the same as subtracting the number)

From the almanac tabular data, at the 14th hour July 15 2001:
GHA table = 28° 30.6'
DEC table = +21° 27.3' N   d = -0.4' (moving less northerly, so -d)

Increment of GHA for the 15 minutes and 37 seconds $Corr^{GHA}$ = 3° 54.3'
GHA = 28° 30.6' + 3° 54.3' = 32° 24.9' = 32.4150°

Increment of DEC for 15 minutes due to rate d is $Corr^{d}$ = - 0.1'
DEC = +21° 27.3' - 0.1' = 21° 27.2' = 21.4533°

Calculations:

Local Hour angle LHA = GHA + Lon = 32.415° + - 67.850° = - 35.435°

Calculated Altitude
Hc = arcSin[Sin(21.453°) · Sin(44.025°) + Cos(44.025°) · Cos(21.453°) · Cos(- 35.435°)]
Hc = 53.0767° = 53° 4.6'

Intercept Offset distance Doffset = 60 · (53.1416° – 53.0767°) = +3.9 n mile
Offset the assumed LOP <u>towards</u> the Sun azimuth.

Calculated Azimuth direction of sun
Zo = arcCos[{Sin(21.453°) – Sin(44.025°) · Sin(53.0767°)}/{Cos(44.025°) · Cos(53.0767°)}]
Zo = 116°, and since LHA is negative (pre-meridian), Zn = Zo = 116°

Moon Shot Example

Let's say this is the data:
DR position Lat = 44.025° N,   Lon = -67.850° W
Eye height = 2 meters   Greenwich date 7/15/2001
Index correction IC = +3.4'
Time of observation UTC = 14h 20m 21s
Sextant measured altitude of the moon Hs = 44° 22.1'   Upper Limb (UL)

Altitude corrections from almanac moon correction tables, in two parts:
$Corr^{DIP}$ = -2.5'
$Corr^{ALT}$ = +50.9' + 3.2' –30.0' (the –30' is for using the UL)   = 24.1'
True altitude Ho = 44° 22.1' + 3.4' -2.5' +24.1' = 44° 47.1' = 44.7850°

From the almanac tabular data, at the 14th hour July 15 2001:
GHA table = 100° 23.7'   v = +12.2'
DEC table = +12° 9.4' N   d = +11.2'   HP = 56.8'

Increment of GHA for the 20 minutes and 21 seconds $Corr^{GHA}$ = 4° 51.3'
Additional increment due to variation v $Corr^{v}$ = 4.2'
GHA = 100° 23.7' + 4° 51.3' + 4.2' = 105° 19.2' = 105.3200°

Increment of DEC for 20 minutes due to rate d is $Corr^{d}$ = +3.8'
DEC = +12° 9.4' + 3.8' = 12° 13.2' = 12.2200°

Calculations:

Local Hour angle LHA = GHA + Lon = 105.32° + - 67.850° = 37.470°

Calculated Altitude
Hc = arcSin[Sin(12.22°) · Sin(44.025°) + Cos(44.025°) · Cos(12.22°) · Cos(37.470°)]
Hc = 44.817° = 44° 49.0'

Intercept Offset distance Doffset = 60 · (44.368° – 44.817°) = -2.0 n mile
Offset the assumed LOP <u>away</u> from the moon's azimuth.

Calculated Azimuth direction of moon
Zo = arcCos[{Sin(12.22 °) –   Sin(44.025°) · Sin(44.817°)}/{Cos(44.025°) · Cos(44.817°)}]
Zo = 123°, and since 0 < LHA <180 (post-meridian), Zn = 360 - Zo = 237°

Star Shot Example

You took a shot of Deneb in the constellation of Cygnus, morning twilight:
DR position Lat = 44.025° N,   Lon = -67.850° W
Eye height = 2 meters   Greenwich date 7/15/2001
Index correction IC = +3.4'
Time of observation UTC = 8h 31m 24s
Sextant measured altitude of Deneb,   Hs = 59° 47.8'

Altitude corrections from the abridged corrections table:
$Corr^{DIP}$ = -2.5'
$Corr^{ALT}$ = -0.5'
True altitude Ho = 59° 47.8' +3.4' –2.5' – 0.5' = 59° 48.2' = 59.8033°

From the almanac tabular data, at the 8th hour July 15 2001:
GHA_{Aries} table = 53° 14.4' SHA_{DENEB} = 49° 37.4'
                                      DEC_{DENEB} = +45° 17.1' N

No *v* or *d* corrections for stars
Increment of GHA for the 31 minutes and 24 seconds $Corr^{GHA}$ = 7° 52.3'
GHA = 53° 14.4' + 7° 52.3' + 49° 37.4' = 110° 44.1' = 110.735°

$DEC = DEC_{DENEB}$ = +45° 17.1' N = +45.2850°

Calculations:

Local Hour angle LHA = GHA + Lon = 110.735° + – 67.850° = 42.885°

Calculated Altitude
Hc = arcSin[Sin(45.285°) • Sin(44.025°) + Cos(44.025°) • Cos(45.285°) • Cos(42.885°)]
Hc = 59.830° = 59° 49.8'

Intercept Offset distance Doffset = 60 • (59.8033° –59.830°) = –1.6 n mile
Offset the assumed LOP <u>away</u> from the star's azimuth.

Calculated Azimuth direction of star
Zo = arcCos[{Sin(45.2850°) – Sin(44.025°) • Sin(59.83°)}/{Cos(44.025°) • Cos(59.83°)}]
Zo = 72°, and since 0 < LHA <180 (post-meridian), Zn = 360 - Zo = 288°

Planet Shot Example

Mars in the evening, same local day, but the next day in GMT:
DR position Lat = 44.025° N,   Lon = -67.850° W
Eye height = 2 meters   Greenwich date 7/16/2001
Index correction IC = +3.4'
Time of observation UTC = 01h 11m 24s
Sextant measured altitude of Mars, Hs = 18° 40.0'

Altitude corrections from the abridged corrections table:
$Corr^{DIP}$ = -2.5'
$Corr^{ALT}$ = -3.0'
True altitude Ho = 18° 40.0' +3.4' –2.5' – 3.0' = 18° 37.9' = 18.632°

From the almanac tabular data, at the 1st hour July 16 2001:
GHA_{MARS} table = 55° 30.6' DEC_{MARS} table = –26° 50.5' S
v = +2.6' and d =0
Increment of GHA for the 11 minutes and 24 seconds $Corr^{GHA}$ = 2° 51.0'
Additional increment due to variation v $Corr^{v}$ = 0.5'
GHA = 55° 30.6' + 2° 51.0' + 0.5' = 58° 22.1' = 58.368°

DEC = DEC_{MARS} = –26° 50.5' = –26.842°

Calculations:

Local Hour angle LHA = GHA + Lon = 58.368° + – 67.850° = – 9.482°

Calculated Altitude
Hc = arcSin[Sin(-26.842°) · Sin(44.025°) + Cos(44.025°) · Cos(-26.842°) · Cos(-9.482°)]
Hc = 18.602° = 18° 36.1'

Intercept Offset distance Doffset = 60 · (18.632° – 18.602°) = +1.8 n mile
Offset the assumed LOP <u>towards</u> Mars's azimuth.

Calculated Azimuth direction of Mars
Zo = arcCos[{Sin(-26.842°) – Sin(44.025°) · Sin(18.602°)}/{Cos(44.025°) · Cos(18.602°)}]
Zo = 171°, and since LHA is negative (pre-meridian), Zn = Zo = 171°

Plots of the Lines Of Position (LOP) from the previous 4 examples

The observer was stationary during all of the observations. The arrows indicate the azimuth direction (bearing from true north) of the heavenly objects. These observations are over the course of a day, from early morning twilight to mid-morning to evening twilight. The ellipse represents the 95% probability area of the position fix using all 4 LOPs.

Best Fix Latitude 43.997

Best Fix Longitude -67.799

Distance from assumed position 2.7 n.miles

LONa -67.85

LOPs from
example problems

Deneb

Sun

LATa 44.025

Mars

Moon
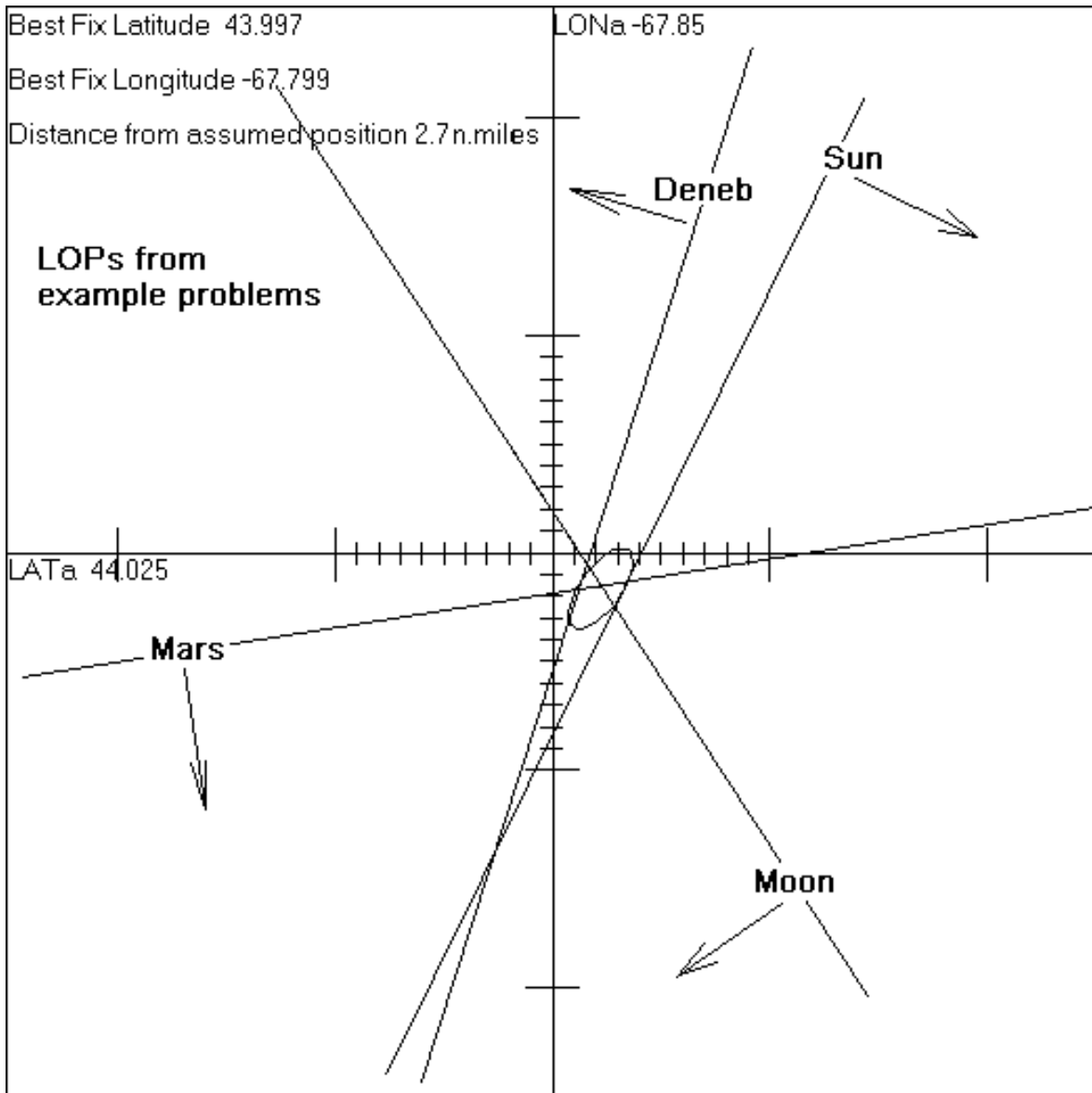
Chapter 9 *Putting it Together and Navigating*

I encourage you the navigator to program your simple calculators to provide the calculated altitude Hc and calculated uncorrected azimuth Zo from inputs of latitude, longitude, GHA, and DEC. It's too easy to make mistakes punching in numbers and doing the trig. A simple programmable calculator mechanizing the simple steps in the calculations will go a long way in reducing the silly arithmetic errors.

Plane Sailing and Dead Reckoning (DR)

With the celestial methods described so far, an important element was the estimated position, also known as the *dead reckoning* (DR) position. Undoubtedly, if you didn't reckon correctly, you would sooner or later regret it. Strictly speaking, an estimated position is not needed, just as it is not needed with the *Global Positioning System*. In the case of GPS, orbiting spacecraft have geographical positions and circles of constant altitude, but electronically it is circles of constant timing. Three spacecraft, three circles and you are pinpointed. But since intersecting straight LOPs is a lot easier than solving simultaneous equations for intersecting circles, an estimated position is essential for our simple methods. In our day-to-day wanderings, flat-Earth approximations are close enough to advance the estimated position from a previously known fix. These approximation methods are known as *plane sailing*.

Dead reckoning is simple to understand on a flat earth, say using your car. If you head northwest at 60 mph, and you drove for 2 hours, you should be 120 miles to the northwest of your last position. But on a spherical surface, the longitude lines start to crowd in on each other as they reach the poles. The 'crowding in' at the current latitude can be thought of as being more or less fixed for short distances. Just think, on the north or South Pole, you could wander across all 360 longitude lines in just a few short steps!
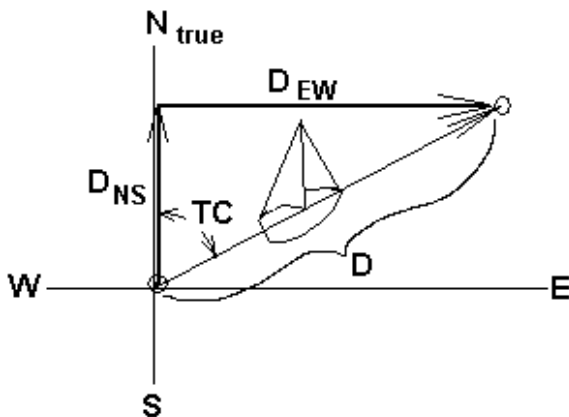
Plane Sailing Shorthand
True course from true north TC
Speed of vessel, knots
Time interval from last fix, hours
$D = \text{Speed} \cdot \text{Time}$ *distance traveled nmile*
$D_{EW} = D \sin(TC)$ *east-west distance*
$D_{NS} = D \cos(TC)$ *north-south distance*
$\Delta Lat = D_{NS}$ *arcmin latitude change*
$\Delta Lon = D_{EW} / \cos(Lat)$ *arcmin longitude change*

Plane Sailing Work Sheet

Last Known Latitude, decimal degrees N+, S-

$$\text{Lat}_O = \quad \longrightarrow \quad \boxed{}$$

Last Known Longitude, decimal degrees E+, W-

$$\text{Lon}_O = \quad \longrightarrow \quad \boxed{}$$

Speed of vessel, corrected for current, knots (kts)

$$V = \quad \longrightarrow \quad \boxed{}$$

Time interval between the present desired fix and the last fix, decimal hours

$$\Delta\text{Time} = \quad \longrightarrow \boxed{}$$

True course made good (heading, compensated for leeway and current), decimal degrees from true north

$$TC = \quad \longrightarrow \quad \boxed{}$$

Estimate of distance, nautical miles (nm)
$$D = V \cdot \Delta\text{Time} \quad\quad\quad \longrightarrow \quad \boxed{}$$

Change in latitude, arcminutes
$$\Delta\text{Lat} = D \cdot \text{Cos(TC)} \quad\quad \longrightarrow \quad \boxed{}$$

New estimated latitude, decimal degrees
$$\text{Lat}_{DR} = \text{Lat}_O + \Delta\text{Lat}/60 \quad \longrightarrow \quad \boxed{}$$

Change in longitude, arcminutes
$$\Delta\text{Lon} = D \cdot \text{Sin(TC)}/\text{Cos}(\text{Lat}_O + \Delta\text{Lat}/120) \longrightarrow \boxed{}$$

New estimated longitude, decimal degrees
$$\text{Lon}_{DR} = \text{Lon}_O + \Delta\text{Lon}/60 \quad\quad \longrightarrow \quad \boxed{}$$

Running Fix

The *running fix* is a method by which two or more line of positions (LOPs) taken at different times on a moving vessel can be coalesced together to represent a navigational fix at any single arbitrary time between the observations. Most frequently, it is used to advance an old LOP to get a fix with a new LOP while the ship is under way. Quite simply, the old LOP is parallel-advanced in the direction of the true course-made-good (TC) to the DR distance between the last LOP and the new one. With a quick study of the figure, the reader should discern the mechanics involved. Essentially, if you produced a 'good' LOP earlier, you can 'drag it' along with your moving vessel as if it were pinned to the stern using the simple distance = rate x time for the distance to drag, and it gets dragged in the same course direction as the vessel.

LOP # 1 at time t_1

LOP #1 advanced to time-coincide with LOP #2

RUNNING FIX

TC

N_T

True Course Direction

W — E

S

LOP # 2 at time t_2

Roffset

Distance-made-good between observations = Speed(Kts) x Time(Hrs)
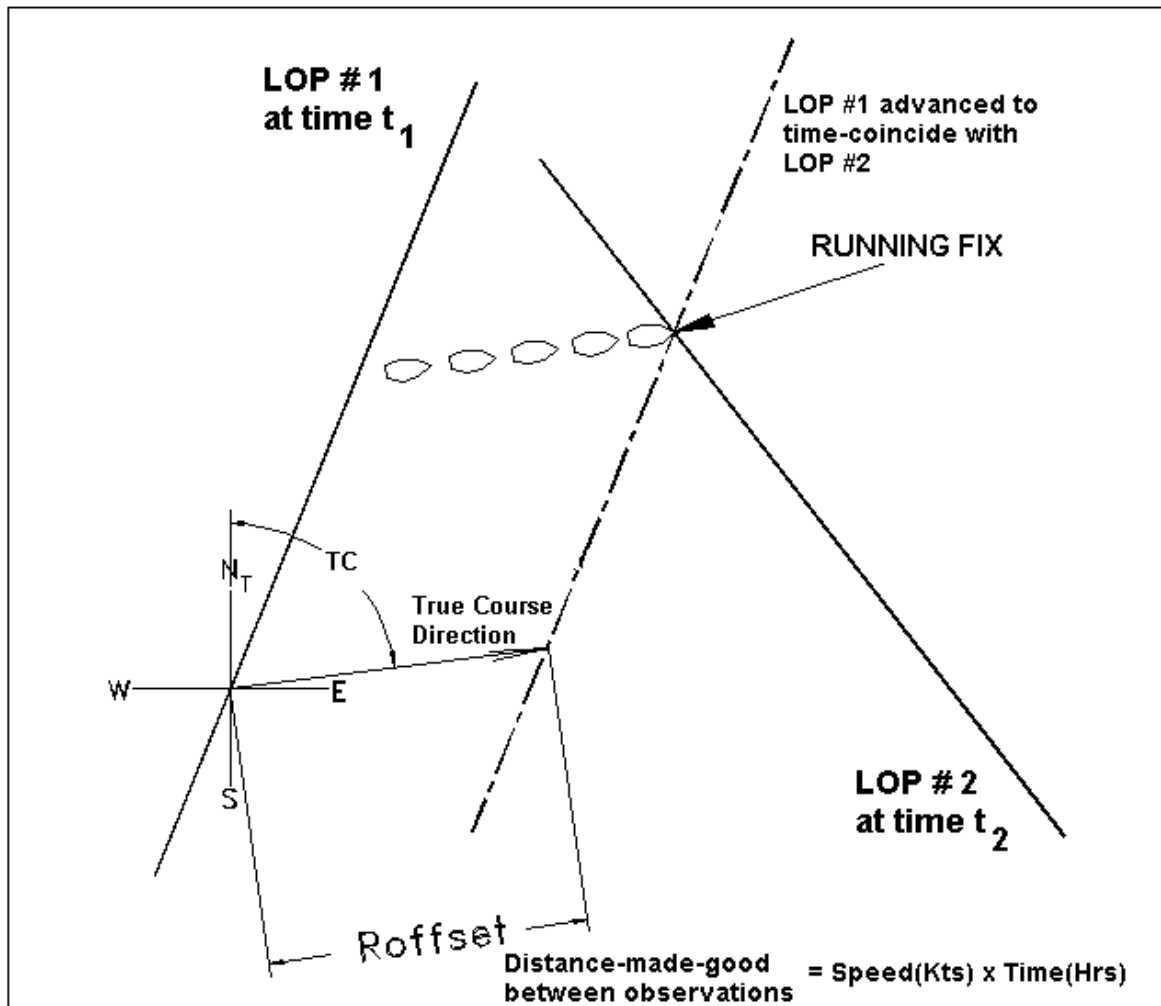
Daily Observation Schedule

During your typical navigating in-the-blue sort of day, you would follow a procedure similar to this:

1) Pre-dawn sighting of planets and stars, providing a definite fix.
2) Mid-morning Sun observation, advancing a dawn LOP for a running fix.
3) Noonish sighting, advancing the mid-morning LOP for a running fix.
4) Mid-afternoon Sun observation, advancing the noon LOP for a running fix.
5) Twilight observation of planets and stars, providing a definite fix.

Note: Morning and evening twilight observations need to be carefully planned. It is a time when both night objects <u>and</u> the horizon are visible simultaneously. That's not a whole lot of time for off-the-cuff navigation. Plan the objects, their estimated altitudes and azimuth angles. Double check with the compass, so that you are sure of what you are looking at.

A Sun-Moon fix is nice when available. When the moon is a young moon, it will be in the sky east of the sun in the late afternoon. When it is an old moon, it will share the sky west of the sun during the morning hours.

65
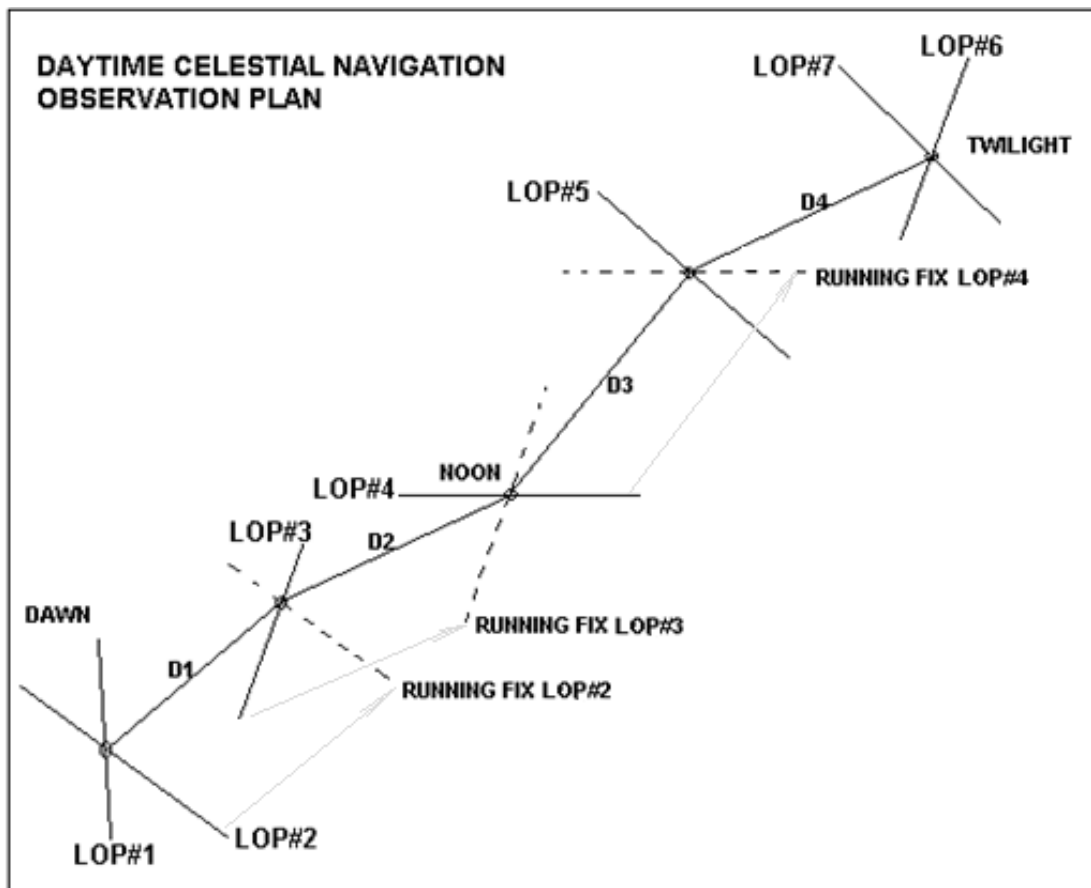
Plotting Multiple Lines of Position (LOP) with Running Fixes

Plotting the LOPs is best done on a **universal plotting** sheet, which is a sheet of paper with a graduated compass rose in the center. This is very convenient, as you can do everything necessary to plot a LOP, requiring in addition a drafting triangle and a scaled ruler. Let us say that we have the true course **TC**, the speed **V** (kts), the times of the observation t_1 , t_2 , t_3 (decimal hrs), etc., the observed altitudes Ho_1, Ho_2, Ho_3, and an assumed position **LATa, LONa**. From sight reduction, we also have the calculated altitudes Ha_1, Ha_2, Ha_3, the intercept distances $Doffset_1$, $Doffset_2$, $Doffset_3$ and the calculated azimuths Zn_1, Zn_2, Zn_3. Since the vessel is continuously underway, we define an arbitrary time that we want the newest fix to apply to. We were at such-and-such location at such-and-such time, even though that time does not correspond exactly to any of the observation times. This selected time for the fix is called the **time of fix**, t_{fix}. We calculate the running fix distance corrections that each observation will require, and designate it $Roffset_1$, $Roffset_2$, $Roffset_3$. The corrections are calculated thus:

$$Roffset_1 = V \cdot (t_{fix} - t_1), \qquad Roffset_2 = V \cdot (t_{fix} - t_2), \text{ etc.. (n.miles)}$$

Notice that for observation times after the time of fix, the offset is negative.

The procedure seems complicated, but after trying it once, the mechanics will seem obvious. Basically you draw the Roffset vector from the center of the compass rose along the direction of the true course, then draw the Doffset vector from the head of the Roffset vector, then draw the LOP from that point.

Here are the detailed steps:

1) The very center of the compass rose on the plotting sheet is designated as the assumed position LATa, LONa. All else is relative to this location.
2) Draw a line thru the center in the direction of the true course TC going both ways, but with an arrow showing the forward direction.
3) Generally there are two scales you can use. The plotting sheet has a built-in scale of 60 n.miles which could just as easily be 6 n.miles for those close encounters. Staying in either the 60 or 6 n.m. scale makes corresponding latitude and longitude measurements possible without calculations.
4) For the first observation, measure along the true course line in the forward direction (if Roffset is +, backwards if Roffset is -) the distance Roffset and mark it with a dot.
5) Then from that mark, draw a line in the azimuth direction Zn, the length being the distance Doffset. If Doffset is negative, draw the line in the opposite direction (180 degrees different). Mark the spot.
6) Draw a line perpendicular to the Zn, passing thru the last mark. This is the LOP compensated for intercept and running to an arbitrary time.
7) Repeat for all the other LOPs.

Plotting a Line Of Position (LOP) on a Universal Plotting Sheet with a running-fix correction Roffset and the intercept offset Doffset
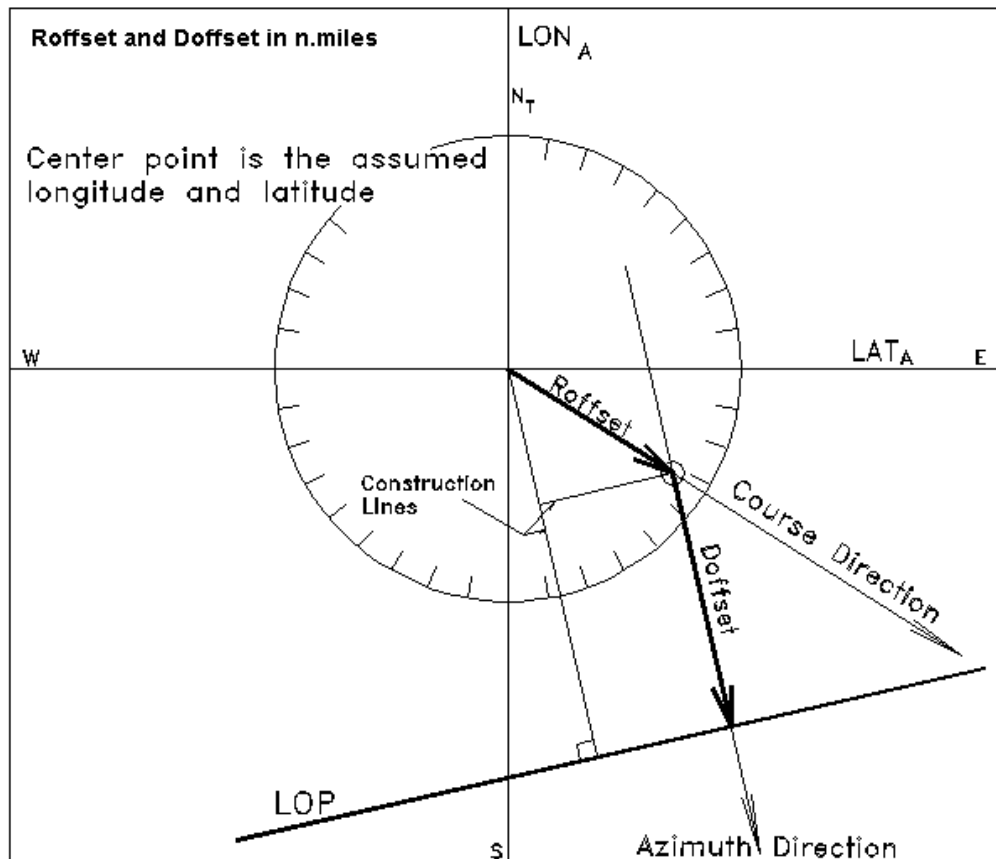
MULTIPLE INTERCEPT LOP with
RUNNING FIX CORRECTIONS

Roffset #1 = V × (t_{fix} − t_1)

Roffset #2 = V × (t_{fix} − t_2)

Roffset #3 = V × (t_{fix} − t_3)

Doffset #1 = 60x (Ho_1 − Hc_1)

Doffset #2 = 60x (Ho_2 − Hc_2)

Doffset #3 = 60x (Ho_3 − Hc_3)

Times of Observation: t_1 t_2 t_3 (hours)

Desired Time of Fix: t_{fix} (hours)

Time t is expressed in decimal hours,
for example, 1.073, 0.374, etc..

Speed V is expressed in knots (n.mi./hr)

Altitude H is expressed in decimal degrees

LOP#1

LOP#2

D_{LON}

Best Fix

LOP#3

Doffset #3

Doffset #2

TC

Assumed Position
LAT LON

Course Direction
at Velocity V

D_{LAT}

Doffset #1

Roffset #3
(if −)

Roffset #2

Roffset #1

Negative (−) Running Fix Correction
(AfterTime of Fix)

Positive (+) Running Fix Correction
(Before Time of Fix)
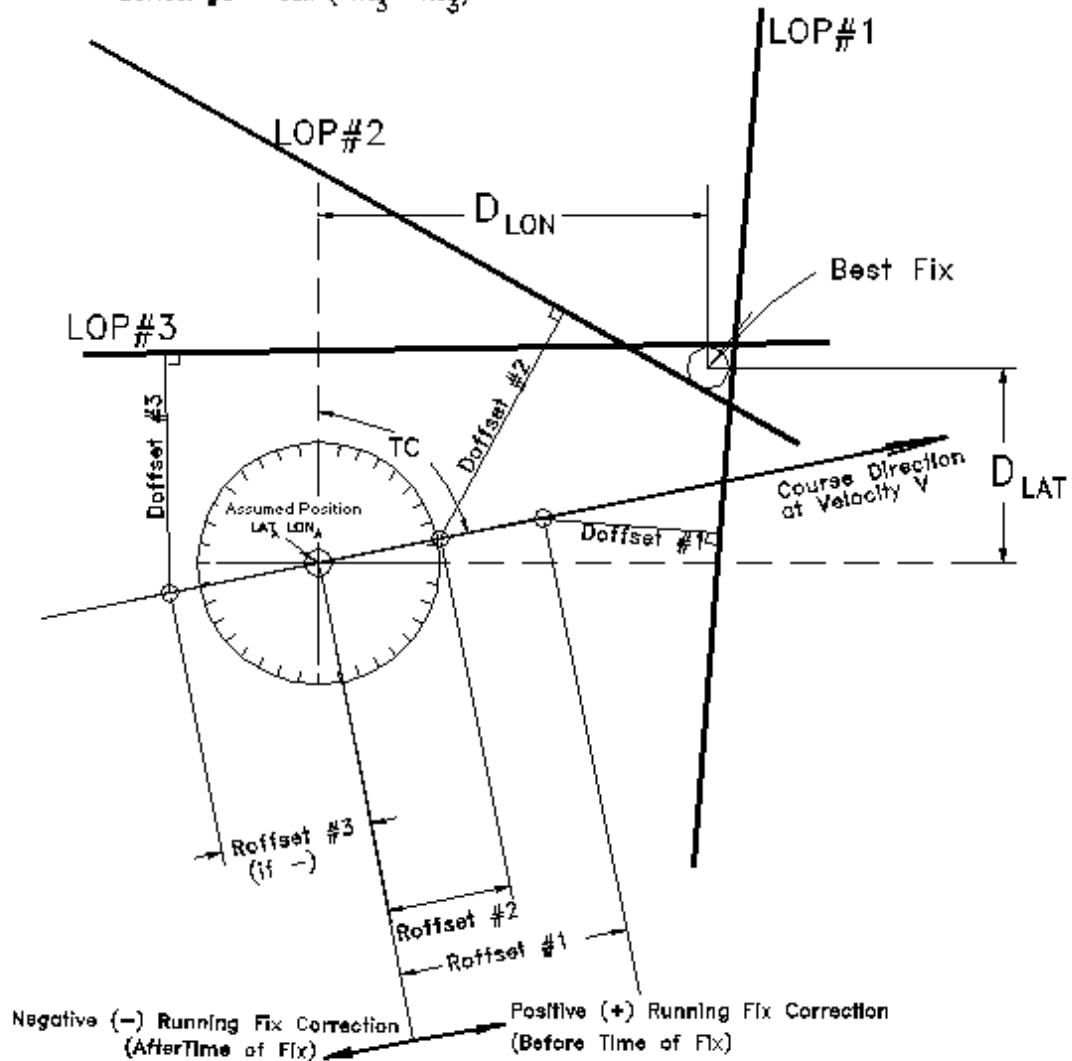
Once all the LOPs are plotted, you can mark what appears to be the best solution of the fix. Measure the distance with the linear scale you are using for the plot from the center. The north-south distance we will designate as $\mathbf{D_{LAT}}$, and the east-west distance as $\mathbf{D_{LON}}$ in nautical miles. Following the sign conventions, if northwards or eastwards, the number is +. If southwards or westwards, the number is -. The corrective change Δ in latitude and longitude from the assumed position **LATa**, **LONa** is thus:

$$\mathbf{LAT_\Delta = (D_{LAT} \; / \; 60)} \qquad\qquad\qquad \text{decimal degrees}$$

$$\mathbf{LON_\Delta = (D_{LON} / \; 60) / \; Cos(LATa + LAT_\Delta/2)} \;\; \text{decimal degrees}$$

The position of the new fix is thus:
$$\mathbf{LAT_{FIX} = LATa + LAT_\Delta}$$
$$\mathbf{LON_{FIX} = LONa + LON_\Delta}$$

The corrective change can also be deduced graphically, from the universal plotting sheet, as it is really set up for this. The compass rose lets you set up your own custom longitude scale for your latitude. Where the latitude angle intersects the circle, you draw the custom longitude line for that position. Remember, 1 nautical mile N-S is equivalent to 1 arc minute of latitude.
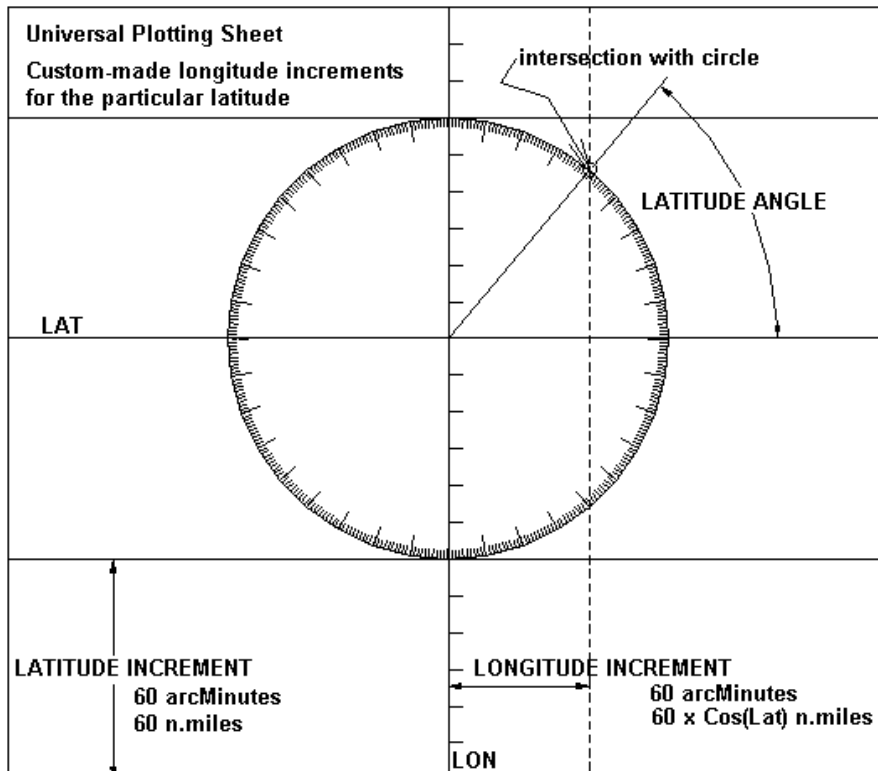
Form the quantities A,B,C,D,E,G from these summations:

$$A = \sum_{n=1}^{N} Cos\left(AZM_n\right)^2 \qquad\qquad B = \sum_{n=1}^{N} Cos\left(AZM_n\right)\cdot Sin\left(AZM_n\right)$$

$$C = \sum_{n=1}^{N} Sin\left(AZM_n\right)^2 \qquad\qquad D = \sum_{n=1}^{N} Cos\left(AZM_n\right)\cdot\left(p1_n + p2_n\right)$$

$$G = A\cdot C - B^2 \qquad\qquad E = \sum_{n=1}^{N} Sin\left(AZM_n\right)\cdot\left(p1_n + p2_n\right)$$

$$\text{Where} \quad p1_n = \frac{Doffset_n}{60} \qquad \text{and} \qquad p2_n = \frac{Roffset_n}{60}\cdot Cos\left(AZM_n - TC\right)$$

$Doffset_n$ is the nth intercept offset distance, n.miles

$Roffset_n$ is the nth running-fix offset distance, n.miles

AZM_n is the nth azimuth direction of the nth heavenly body

TC is the true course angle from true north

$$LON_I = LON_A + \frac{(A\cdot E - B\cdot D)}{G\cdot Cos\left(LAT_A\right)} \qquad \text{Improved Longitude estimate from the assumed position}$$

$$LAT_I = LAT_A + \frac{(C\cdot D - B\cdot E)}{G} \qquad \text{Improved Longitude estimate from the assumed position}$$

$$dist = 60\cdot\sqrt{\left(LON_I - LON_A\right)^2\cdot cos\left(LAT_A\right)^2 + \left(LAT_I - LAT_A\right)^2}$$

Distance from assumed fix to calculated fix, nm. Should be < 20 nm.
If not, use the improved fix as the new assumed position and start all over again

Good Practice and Error Reduction Techniques

There are many sources of error, not the least misidentification of the heavenly object. Slim chance of that happening with the Sun or Moon. With objects that you are sure of, a set of 3 or 4 shots of each known object can reduce random measurement errors. With stars at twilight, perhaps it is better to take single shots but have many targets to reduce the effects of misidentification. This type of error has the distinction of putting you hundreds of miles off, and so are easy to catch, allowing you to disregard the specific data.

Handling measurement random errors graphically:

Graphically averaging

Graphically averaging with slope
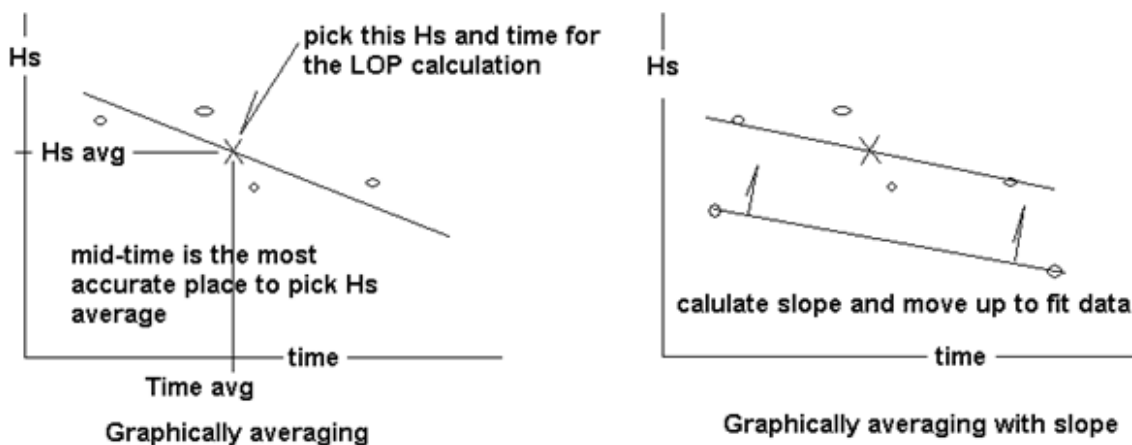
Random Errors
The effect of multiple shots of the same object are such that the random errors, some +, some -, will average to zero. Random measurement errors of plus or minus several miles are handled several basic ways for a set of shots of the same object:
1) Calculate all the LOPs in a set and average them graphically on the map.
2) Arithmetically average the times and altitudes for a set of shots, and calculate one LOP using the averaged value of time and altitude.
3) Graph the set of shots with time on the horizontal and altitude on the vertical. Draw a line representing the average and from that pick one time and altitude from the line to calculate one LOP. This is the easiest.
4) Graph the shots as in 3), but calculate a slope and fit it to the data. The slope is determined by calculating Hc for two different times in the range of the data set with your estimated position. With these two new points, draw a line between them. Parallel offset this new line until it fits best in the data points already drawn. Then, as in 3), pick one time and altitude from the line to calculate one LOP.

Using technique 4) should result in the most accurate LOP; however there are more calculations making it the same trouble as 1). On the other hand, practical navigation is not usually concerned with establishing a position to within ¼ mile, so unless you are particular, **graphing your measurements as in technique 3) may be the easiest to implement with a good payoff for reducing random errors**. Arithmetically averaging instead of graphically averaging is a good way to introduce unwanted calculation mistakes, so I would steer away from technique 2) for manual calculations.

Systematic Errors
This species of error, where a constant error is in all of the measurements, can come from such things as an instrument error, a misread index error, your personal technique and bias, strange atmospheric effects, and clock error. All but clock error can be handled with the following technique. If you have many objects to choose from, choose 4 stars that are ~90 degrees apart from each other in azimuth, or with 3 stars make sure they are ~120 degrees apart. This creates a set of LOPs where the effect of optical systematic errors cancels.

Examples of Systematic Error:

LOPs 90°apart reveal a constant offset from the true position
Random error
Systematic error
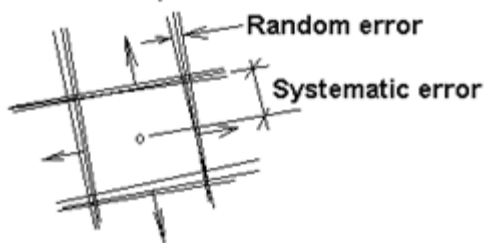
LOPs 60°apart falsely implying position x with systematic error
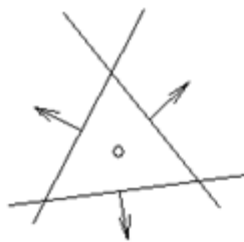
LOPs 120°apart canceling systematic error

The steps for good practice

1) For a good fix, pick 3 or more <u>clearly</u> identified heavenly objects.

2) Pick objects that are spaced in azimuth 90 to 120 degrees apart for systematic error reduction.

3) If you can, make a tight spaced grouping of 3 shots per object.

4) Apply averaging techniques for random error reduction.

5) Advance the LOPs with a running fix technique to time coincide with the time of your last shots.

Chapter 10   *Star Identification*

There are various star finding charts, the 2102-D and Celestaire star chart come to mind. However, you could use the equations for calculated altitude and azimuth, rearranged, to help you identify stars. Now this only applies to the bright 58 'navigational stars', as data for their position on the celestial sphere (star globe) is given.
Rearranging the azimuth equation, we get the declination DEC:

DEC = arcSin[cos(AZM) · cos(Lat) · cos(H) + sin(Lat) · sin(H)]

If the declination is +, it is North, if – then it is South.
AZM is the approximate azimuth angle (magnetic compass + magnetic variation), Lat is the assumed latitude, and H is the altitude angle (don't bother with dip and refraction corrections).
Rearrange the calculated altitude equation to get local hour angle LHA:

LHA = (+/-) arcCos[{sin(H) - sin(DEC) · sin(Lat)} / {cos(Lat) · cos(DEC)}]

If the azimuth is greater than 180°, then LHA is **+**.
If the azimuth is less than 180°, then LHA is **–**.
The sidereal hour angle ('longitude' on the star globe) is then:

SHA = LHA – GHA$_{Aries}$ – Lon

Where GHA aries is the Greenwich hour angle of aries (zero 'longitude' on the star globe) at the time of this observation from the almanac, and Lon is the assumed longitude position.

Once you have the essential information, SHA and DEC, then you can look it up in the star chart data to match it with the closest numbers. If the numbers still don't match any stars, then look in the almanac to match up SHA and DEC with any planets listed.

Star magnitudes refer to their brightness. Numerically, the larger the number, the dimmer the star. The brightest stars actually have negative magnitudes, such as Sirius (the brightest star) has a magnitude of -1.6.

It's actually best to this by software, or have constellation charts handy.

The 58 Navigational Stars Listing

SHA	DEC	Star	magnitude	color	constellation
358	+29	Alpheratz	2.2		Great Square
354	-42	Ankaa	2.4	Orange	Phoenix
350	+57	Schedar	2.5	Orange	Cassiopeia
349	-18	Diphda	2.2		Cetus
336	-57	Achernar	0.6		Eridanus
328	+23	Hamal	2.2	Orange	Aries
324	+89	Polaris	2.1		Little Dipper
315	-40	Acamar	3.1		Eridanus
315	+4	Menkar	2.8		Cetus
309	+50	Mirfak	1.9		Perseus
291	+16	Aldebaran	1.1	Orange	Taurus
281	-8	Rigel	0.3	Blue	Orion
281	+46	Capella	0.2	Yellow	Auriga
279	+6	Bellatrix	1.7		Orion
279	+29	Elnath	1.8		Taurus
276	-1	Alnilam	1.8		Orion
271	+7	Betelgeuse	1.0	Red	Orion
264	-53	Canopus	-0.9	White	Carina
259	-17	Sirius	-1.6	White	Canis Major
255	-29	Adhara	1.6		Canis Major
245	+5	Procyon	0.5	Yellow	Canis Minor
244	+28	Pollux	1.2		Gemini
234	-59	Avior	1.7		Carnia
223	-43	Suhail	2.2		Vela
222	-70	Miaplacidus	1.8		Carnia
218	-9	Alphard	2.2		Hydra
208	+12	Regulus	1.3		Leo
194	+62	Dubhe	2.0		Big Dipper
183	+15	Denebola	2.2		Leo
176	-18	Gienah	2.8		Cygnus
173	-63	Acrux	1.1		S. Cross
172	-57	Gacrux	1.6		S. Cross
167	+56	Alioth	1.7		Big Dipper
159	-11	Spica	1.2	Blue	Virgo
153	+49	Alkaid	1.9		Big Dipper
149	-60	Hadar	0.9		Centarus

SHA	DEC	Star	magnitude	color	constellation
148	-36	Menkent	2.3		Centarus
146	+19	Arcturus	0.2	Orange	Bootes
140	-61	Rigil Kent	0.1		Centarus
137	-16	Zuben'ubi	2.9	White	Libra
137	+74	Kochab	2.2	Orange	Little Dipper
126	+27	Alphecca	2.3		Corona Borea.
113	-26	Antares	1.2	Red	Scorpio
108	-69	Atria	1.9		S. Triangle
103	-16	Sabik	2.6		Ophiuchus
097	-37	Shaula	1.7		Scorpio
096	+13	Raselhague	2.1		Ophiuchus
091	+51	Eltanin	2.4		Draco
084	-34	Kaus Australis	2.0		Sagittarius
081	+39	Vega	0.1	White	Lyra
076	-26	Nunki	2.1		Sagittarius
062	+9	Altair	0.9		Aquila
054	-57	Peacock	2.1		Pavo
050	+45	Deneb	1.3		Cygnus
034	+10	Enif	2.5	Orange	Pegasus
028	-47	Al Na'ir	2.2		Grus
016	-30	Fomalhaut	1.3		Piscis Austrin.
014	+15	Markab	2.6		Great Square

If one waits until the Little Dipper is in this orientation, then the 41' that Polaris is off from the celestial pole won't matter and then the observed altitude will equal the latitude. Kochab will be slightly lower than Polaris.

THE NAVIGATIONAL STARS

Courtesy of Celestaire Inc.

Capella

Auriga

7 Sisters

Elnath

Tarus

Pollux

Aldebaran

Gemini

Bellatrix

Orion

Betelgeuse

Canis Minor

Alniliam

Procyon

Rigel

Sirius

The "summer triangle"

Chapter 11 *Special Topics*
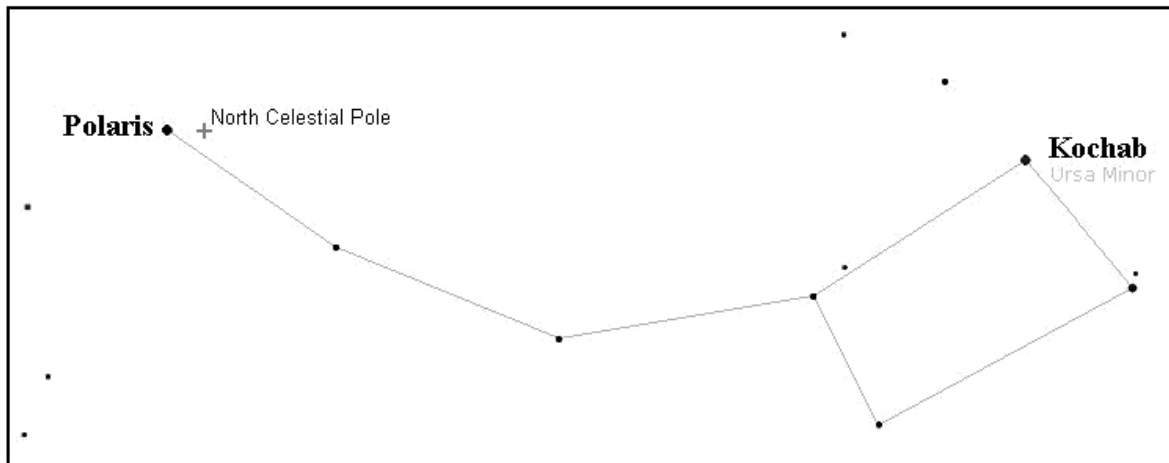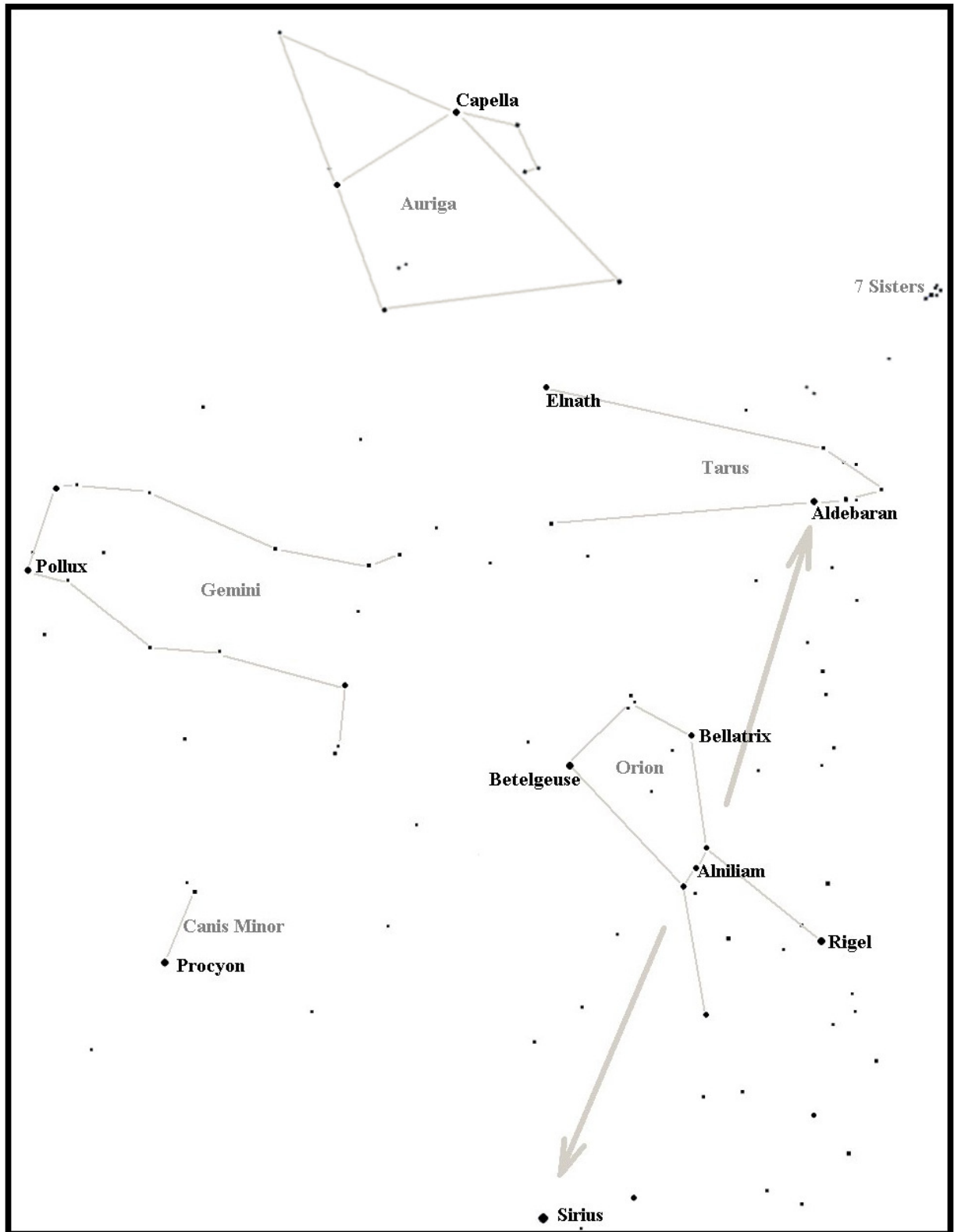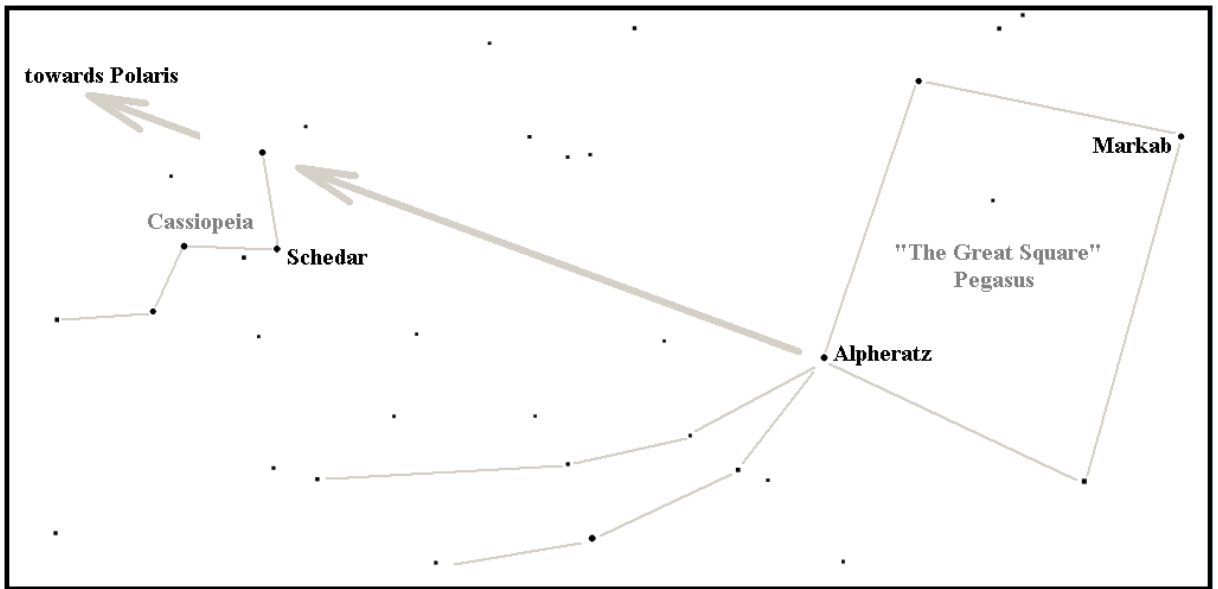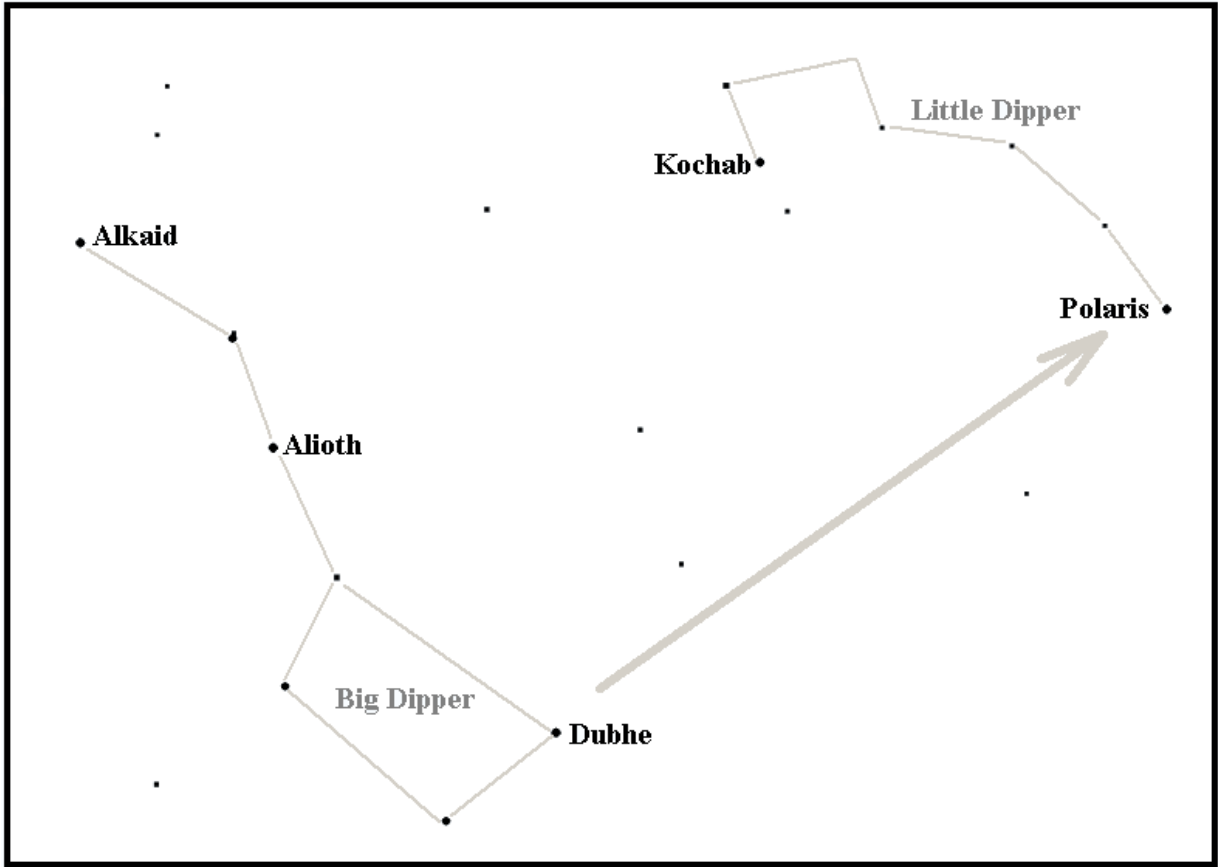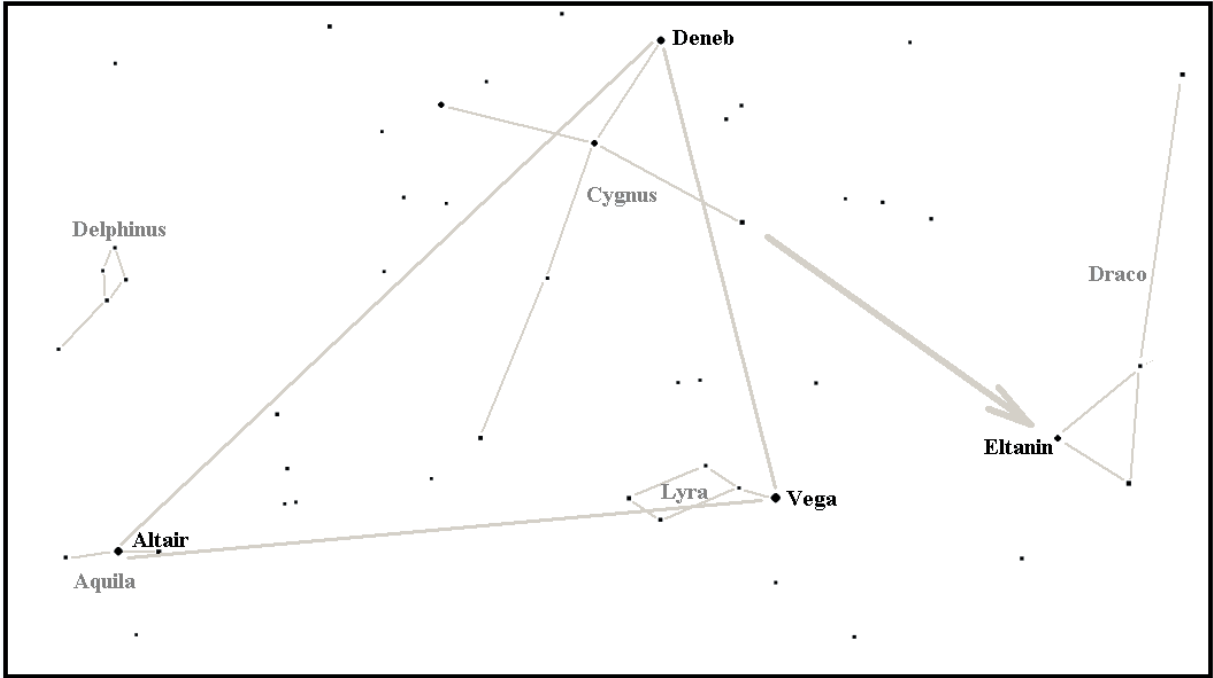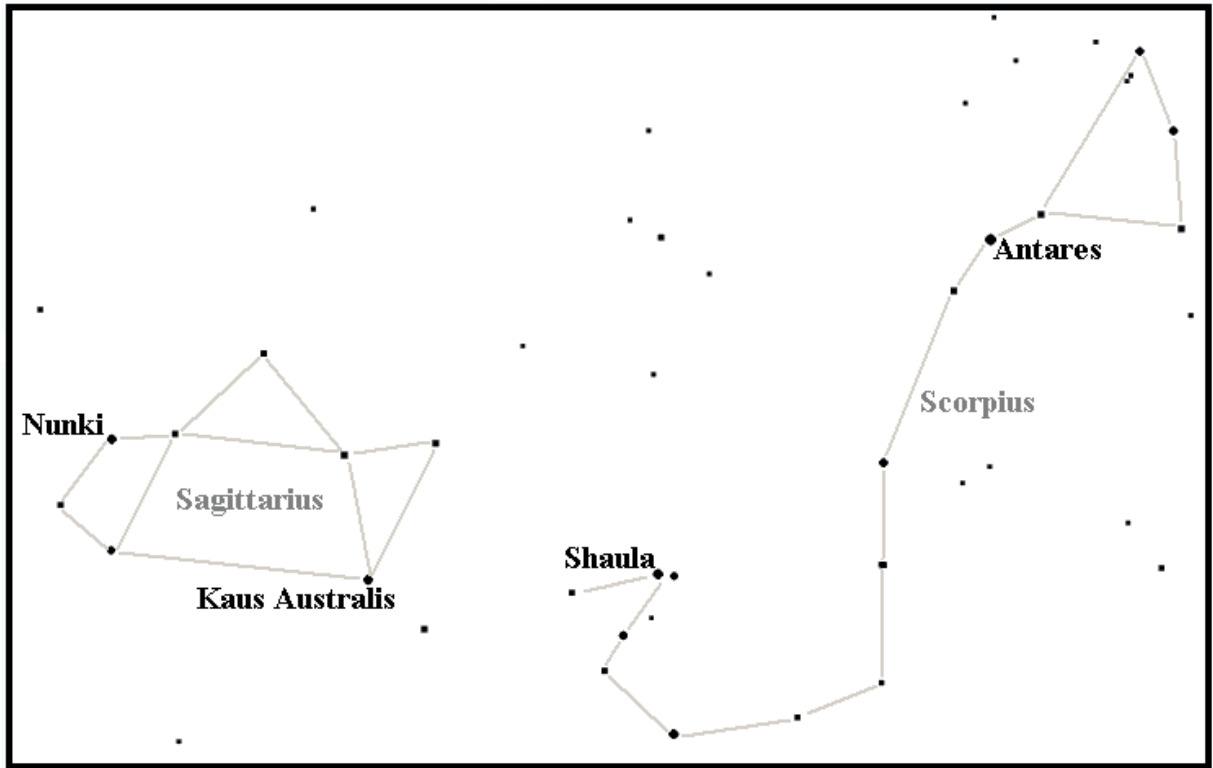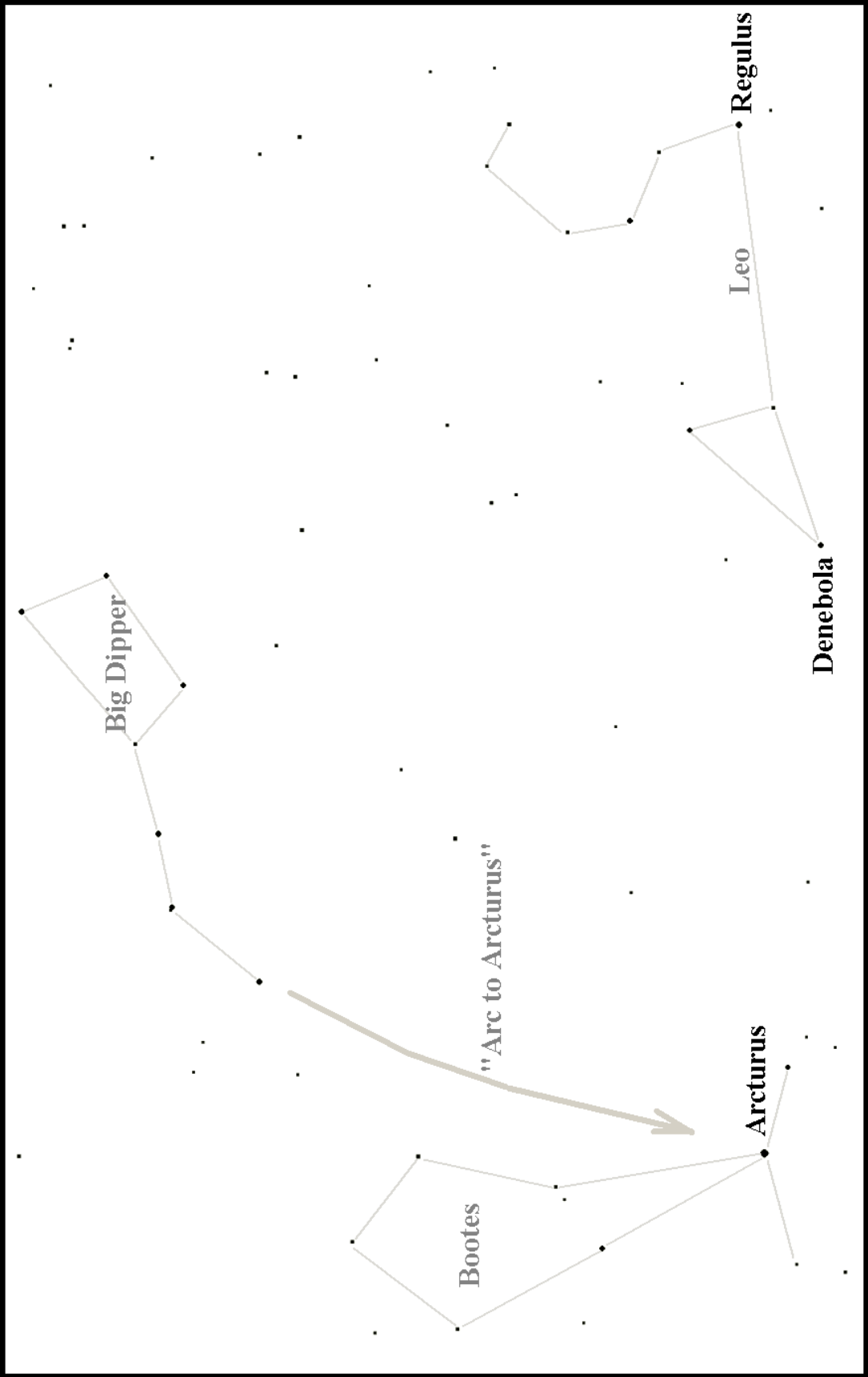
Determining Longitude and Latitude Individually
Some simplified methods can be used at specific times of the day to calculate latitude and longitude individually, then combining them with running fix techniques. A scenario like this presents itself: take the height of Polaris at dawn twilight for a latitude fix, then with the timing of sunrise or just after with the *prime vertical sight* determine longitude. Use the running fix technique to 'forward' along the latitude LOP to time coincide with the longitude LOP.

These techniques are probably not used so much anymore, since with tabular methods and calculators the complexity of the navigational triangle is not so daunting. In other words, the only limitations now are ones of visibility of the heavenly object, not mathematical.

Latitude Determination, a purely East-West LOP

By meridian transit
This has already been covered in the discussion of the noon sighting for the sun. One could do it for any heavenly body, but the sun is the favorite.

By the height of Polaris
Since Polaris is not exactly on the celestial north pole, corrections for this slight offset and annual aberration must be accounted for. The nautical almanac has tables where:
Latitude = Ho -1° + ao + a1 + a2, where Ho = Hs + IC + Corr^{DIP} + Corr^{ALT}
ao is a function of local hour angle LHA
a1 is a function of estimated latitude
a2 is a function of what month it is

By the length of time of day
If you measure the time of day from sunup to sunset in hours, minutes, and seconds, you can calculate your latitude. Start the timing and end the timing when the sun's lower limb is about ½ diameter above the horizon. Convert the time into decimal hours, and name it ElapsedTime.

Lat = arcTan[-cos(7.5 · ElapsedTime) / tan(DEC)]

Since there are refraction effects, use the altitudes calculated in the next section.

Longitude Determination, a purely North-South LOP

By the timing of sunrise or sunset
The equations simplify when the true altitude Ho is zero. But due to dip, refraction, semi-diameter and index error, the sextant altitude needs to be pre-set at a low but specific angle to catch the sun at true horizon sunrise or sunset. If:
$Ho = Hs + IC + Corr^{DIP} + Corr^{ALT}$ then $Hs = Ho - IC - Corr^{DIP} - Corr^{ALT}$
So, for Ho = 0:
$Hs = - IC - Corr^{DIP} - Corr^{ALT}$
It should be apparent that:
$Ha = - Corr^{ALT} = -R - SD_{LL}$ or $= -R + SD_{UL}$
Since the average sun semi-diameter is 16', we can figure the refraction correction for when Ho = 0. Refraction correction is a function of Ha, so we need to do a little iteration. Fortunately I've done it for you, so here are the results:
Using the sun's lower limb (LL), the $Corr^{ALT} = - 15.5$'   LL
Using the sun upper limb (UL), the $Corr^{ALT} = - 43$'   UL

In short, set the sextant to:
$Hs = - IC - Corr^{DIP} + 15.5$' (LL) or $Hs = - IC - Corr^{DIP} + 43.0$' (UL)

The dip correction is always negative, but in this equation the double negative will make this number a positive. Same with the altitude corrections in this case. With the sextant preset to this angle, when the sun's limb kisses the horizon, observe the time UTC. In the almanac, look up the GHA and declination, adding the increments for the minutes and seconds. Longitude is then:
Lon = {(+ / -) arcos[-Tan(Lat) · Tan(DEC)]} - GHA,
 (+ / -) negative if sunrise or positive if sunset

Example:
IC = -2.1', h = 2 meters, so $Corr^{DIP}$ = - 0.5'.   Latitude = 41.75°
With the sun's LL $Corr^{ALT}$ = - 15.5'
So, preset the sextant angle to Hs = -(-2.1') – (-0.5') – (- 15.5') = + 0° 18.1'
When the sun is at this altitude, the time was 11h 30m 10s. From the almanac let's say that GHA =345.390°, and DEC = 10.235° N
So: Lon = – arcos[-Tan(41.75°) · Tan(10.235°)] - 345.39°   = - 444.664°
Add 360 to it, Lon = 360° – 444.664° = - 84.664° West Longitude

By the prime vertical sight
If you will recall the illustration on page 21, the prime vertical circle goes from due east to the zenith to due west. In the summer months, the sun will rise a bit to the north of east (northern hemisphere) and it may be some time after sunrise that the sun crosses this imaginary line. When it does, the azimuth is exactly 90°. This simplifies the equations such that:

Ho = arcsin[Sin(DEC) / Sin(Lat)]
Work out the sextant angle by:
Hs = Ho – IC – Corr^{DIP} – Corr^{ALT}
Determine the UTC time when this condition occurs by waiting for the object to attain Hs, then look up in the almanac GHA for the sun. Then:
Lon = (+ / -) arcsin[Cos(Ho) / Cos(DEC)] - GHA
(+ / -) negative if sunrise or positive if sunset

Example:
IC = -2.1', h = 2 meters, so Corr^{DIP} = - 0.5'.   Latitude = 41.75°
If Ha ~ 15° then Corr^{ALT} = +12.5'
For the approximated time, from the almanac DEC = 10.260° N
Ho = arcsin[Sin(10.260°) / Sin(41.75°)] = 15.515°
When the sun is at this altitude, the time was 12h 54m 3s. From the almanac let's say that GHA =6.365°, and DEC = 10.260° N
Lon = - arcsin[Cos(15.515°) / Cos(10.260°)] – 6.365° = - 84.664° West

By the time sight
This uses the Sumner line equation, used only once by imputing your best estimate for latitude. Be careful of the (+/-) sign, determine if the object is pre or post meridian. Easily done with the sextant, if the object continues to rise, it is pre meridian. The closer to meridian transit the less accurate the answer since at meridian transit the LOP is East-West, not North-South. In these circumstances, a little error in latitude will translate into a large longitude error from the calculation.
East side of the circle when the object is westwards (post meridian):
Lon = arcCos[{ Sin(Ho) - Sin(DEC) · Sin(Lat)}/{Cos(Lat) · Cos(DEC)}] – GHA

West side of the circle when the object is eastwards (pre meridian):
Lon = -arcCos[{ Sin(Ho) - Sin(DEC) · Sin(Lat)}/{Cos(Lat) · Cos(DEC)}] – GHA

By the haversine method

One can do this straight with a calculator, or use Norie's tables which have log values and haversines to turn multiplication into addition. That is the basic application of logs, such that if you convert each value of a multiplier by its log, then you can add up the results, then finally inverse log to get the final product. For example, if an as yet uncalculated quantity d = a x b x c, then when converted it becomes log(a) + log(b) + log(c) = log(d). Add up the three logs then de-convert the result to get d. The point is that addition is easier on the sailor's brain than multiplication!

Define the haversine function hav of angle a: hav(a) = (1 – cos(a)) / 2
HA = Hour Angle = absolute value of LHA = |LHA|
ZX = co-altitude = 90 – Ho
Lat is latitude   Dec is declination of the object.

Basic haversine formulation based on navigational triangle:
hav(ZX) =   hav(Lat – Dec)   +   cos(Lat) ·cos(Dec) ·hav(HA)

We algebraically manipulate this so that HA is alone on the left side of the equation:
hav(HA) =   [hav(ZX) - hav(Lat – Dec)] / [cos(Lat) · cos(Dec)]

With the simple definition of secant(a) = sec(a) = 1/cos(a):
hav(HA) =   [hav(ZX) -   hav(Lat – Dec)] · sec(Lat) · sec(Dec)]

Taking the log of both sides, and using the rules of log where multiplication is converted to addition:

Log[hav(HA)] =
Log[hav(ZX) - hav(Lat – Dec)]   +   Log[sec(Lat)]   +   Log[sec(Dec)]
Define   havDiff =   [hav(ZX) - hav(Lat – Dec)]

So finally:
Log[hav(HA)] = Log[havDiff] + Log[sec(Lat)] + Log[sec(Dec)]

Since we have ZX, Lat, and Dec, we can add up the quantities on the right, which then equals the Log of the Haversine of HA. Then we convert log back to the Haversine, then convert the Haversine back to just the angle HA. If you have a special table you can do the double conversion in one step.

Assume we have tables for Haversine of angles, log of quantities, log secant of angles, and log Haversine of angles.

In an algorithm it would look like this:

1)   Determine latitude Lat from a noon sighting not too many hours ago and even update to a DR Lat with plane sailing calculations

2)   Take a sun shot at time T where you measure the sextant angle Hs

3)   Convert Hs to Ho using appropriate corrections

4)   Determine the GHA of the sun at time T with the Nautical Almanac

5)   Determine co altitude $ZX = 90 - Ho$

6)   Determine latitude difference between you and the sun (Lat – Dec), paying attention to the signs for Lat and Dec (N is +, S is -). Take the absolute value of the final difference (no negative numbers)

7)   With table, Natural Haversine of ZX

8)   With table, Natural Haversine of (Lat – Dec)

9)   Subtract 8) from 7); Natural Haversine -   = havDiff = hav(ZX) - hav(Lat – Dec)   = 7) minus 8)

10) With table Convert to Log = Log[havDiff]

11) With table L. Sec. DR. Lat = Log[sec(Lat)]

12) With table L. Sec. Dec = Log[sec(Dec)]

13) Add the last three quantities; Log. Hav. HA = 10) + 11) + 12) = Log[havDiff] + Log[sec(Lat)] + Log[sec(Dec)]

14) With log Haversine table, convert quantity 13) back to an angle HA

15) If post-meridian subtract HA from GHA, if pre-meridian then add to GHA;   Lon = 360 – (GHA +/- HA)

Here is a MathCad implementation of the above haversine method with an example from a voyage in 1977 by Jack & Jude, an adventurous Australian couple, somewhere in the Indian Ocean. The latitude was determined first with a noon sighting, then several hours later another sun shot was taken for the longitude calculation.

$$H_o := \left(50 + \frac{42}{60}\right) \qquad Lat := \left(5 + \frac{38}{60}\right) \; N \qquad Dec := -\left(8 + \frac{19.7}{60}\right) \; S \qquad GHA := \left(307 + \frac{24.4}{60}\right)$$

$$ZX := 90 - H_o \quad = \quad \left(39 + \frac{18}{60}\right) \qquad\qquad (Lat - Dec) = \left(13 + \frac{57.7}{60}\right)$$

Feb 27, 1977

Nat. Hav. ZX $\qquad\qquad hav(ZX) = 0.11308$

Nat. Hav. DR. Lat-Dec $\qquad hav(Lat - Dec) = 0.014771$

Nat. Haversine - $\qquad\qquad hav_{diff} := hav(ZX) - hav(Lat - Dec) \qquad hav_{diff} = 0.098309$

Convert to Log $\qquad\qquad \log\left(hav_{diff}\right) = -1.007409$

L. Sec. DR. Lat $\qquad\qquad \log(sec(Lat)) = 0.002103$

L. Sec. Dec. $\qquad\qquad \log(sec(Dec)) = 0.004604$

Log. Hav. H.A. = $\qquad\qquad \log\left(hav_{diff}\right) + \log(sec(Lat)) + \log(sec(Lat)) = -1.003$

$$HavHA := 10^{LogHavHa} \qquad\qquad HavHA = 0.099265 \qquad \text{With no table, this}$$
$$\text{is done in 2 steps}$$

$$HA := 2 \cdot asin\left(\sqrt{HavHA}\right) \quad = \quad \left(36 + \frac{43.7}{60}\right) \qquad \text{If post meridian shot, subtract LHA from GHA}$$
$$\text{If premeridian shot, add LHA to GHA}$$

$$\begin{array}{c}(GHA - HA) \\ \text{post meridian}\end{array} \quad = \quad \left(270 + \frac{40.6}{60}\right)$$

Observed Long. $\qquad 360 - (GHA - HA) \quad = \quad \left(89 + \frac{19.4}{60}\right)$

The observation was post-meridian, so HA is subtracted from GHA.

Chapter 12   *Lunars*

These days, with quartz watches and radio time-ticks, lunars are for the hard-core celestial zealot. This is a method whereby you can reset your untrustworthy chronometer if you are in the middle of the ocean (or anywhere) without friends or a short-wave radio. Or perhaps you just want to feel challenged.

Essentially the arc-distance between the moon's limb and a heavenly object close to the ecliptic plane (such as a planet) is measured. Since the arc distance is changing with time relatively fast (~0.508 deg per hour), one can infer a particular time in UT to a particular arc distance. The nautical almanac contains predictions for both objects, and so the arc distance between the two objects can be worked out as a function of time. The almanac many years ago contained these functions, but stopped in 1907. It must be done by calculation or by special lunar tables.

Since the moon appears to orbit about the Earth once every 29 ½ days (27 1/3 days in inertial space), the angular closing speed between the moon and a planet or star near the ecliptic plane, from our earthly point of view, is about <u>0.5 arcminute per minute of time</u>. Practically speaking, between messy observations and even messier calculations, this means you won't get any closer to the real time by a minute or so. Still, that's not bad, it just means you'll have to make allowances in your longitude estimate to the tune of $15 \cdot \text{Cos(Lat)}$ n.miles per minute of time error. But you won't know the error, so you'll just have to assume something like 2 minutes of time.

The tabular data in the almanac does not consider refraction or parallax, and so the observer will have to correct for it. In order to do that, the observer must nearly simultaneously obtain the altitudes of both the moon and star (or planet) as well as the actual measured arc distance between the two. Whew! It helps to have two friends in the same boat with sextants. It is possible that the errors will be small if you take three consecutive measurements within a few minutes, since the altitude measurements are for refraction and parallax *corrections*, which won't change fast. By small, I mean the time estimate may be off by several minutes per degree of altitude change. A degree of altitude change at it's worst will take 4 minutes (at the equator). But if the measurements are taken with the objects near the meridian line, you may have quite a bit of time to make measurements sequentially. In fact, one can measure sequentially and correct the altitude measurements to time coincide with the arc distance measurement, a sort of 'running fix' correction on altitude.

If the time difference between the arc measurement and altitude measurement is **δ**T minutes of time, then add this increment to the altitude measurement:

δH = 15 · [-Cos(LAT) · Cos(DEC) · Sin(LHA)/Cos(Ho)] · δT arcminutes

where **δT = Tarc – Taltitude** in minutes of time. **Tarc** refers to the time you took the arc distance measurement, and **Taltitude** is the time you took the altitude measurement. The absolute time is not important; rather the time difference is what should be accurate. Since there are 2 altitude measurements, there will be a **δH**star, and **δH**moon increment based on time increments **δT**star, **δT**moon.

LAT is your latitude, DEC is the declination of the observed object, LHA is your best guess at the local hour angle for the object, and Ho is the observed altitude for the object (Hs + SD is close enough). This way, the parallax and refraction corrections will be identical had you done simultaneous measurements.

When the measured arc distance **Ds** is corrected for index error, refraction, and semi-diameter, it is referred to the apparent arc distance **Da**. When final corrections are made for parallax, the resulting number is the arc distance as seen from an observer at the Earth's center. That final arc distance is designated as **D**cleared and the entire procedure is known as **clearing the lunar distance**. The equation for **D**cleared presented here was first published in 1856 by J.R. Young, although I derived the exact same equation independently when approaching the problem.

The case presented is for when you don't know the exact time and you have made the three necessary measurements as though you were doing it for real. Besides, it's fun. Well, sort of. This entire task is simplified if you have a computer and use MathCad software to write and evaluate the equations. By the way, good luck. Oh, as far as sequencing the observations to minimize errors if you don't feel like making the 'running fix' corrections, do this:

   1) measure the arc distance first
   2) measure the altitude of the most east/west next, quickly
   3) measure the altitude of the southern/northern most object last

The objects farthest away from meridian passage change altitude the quickest and should be measured soonest after the arc distance measurement.

There is a second method using one measurement and an assumed position. Here the method assumes that the latitude assumed is very accurate, but the longitude is as far off as the chronometer. Employed best by computer.

LUNARS, ILLUSTRATION OF ANGLES
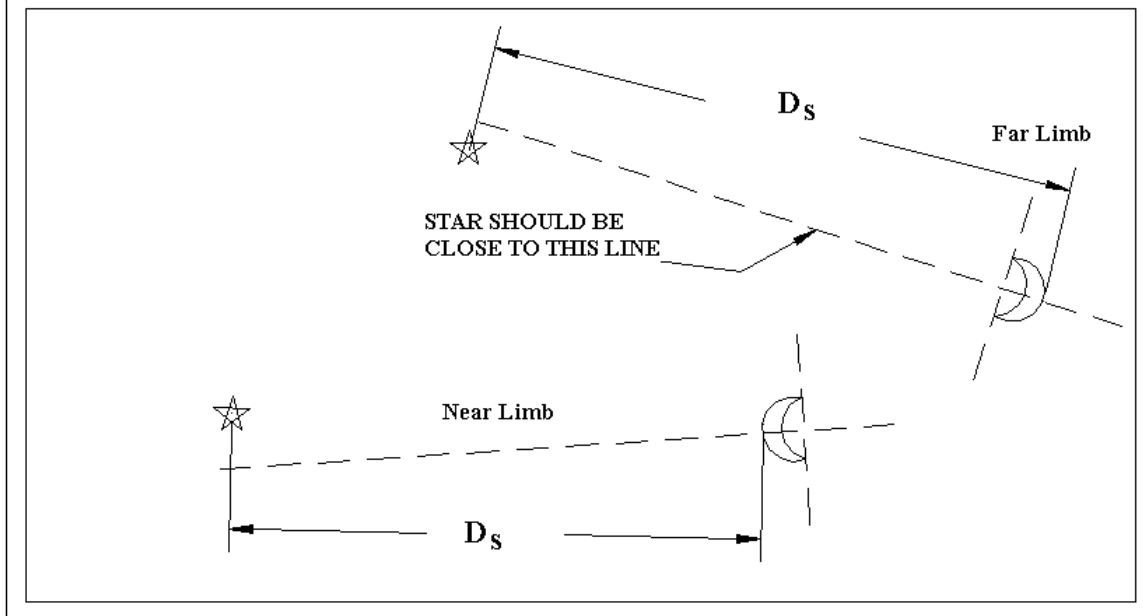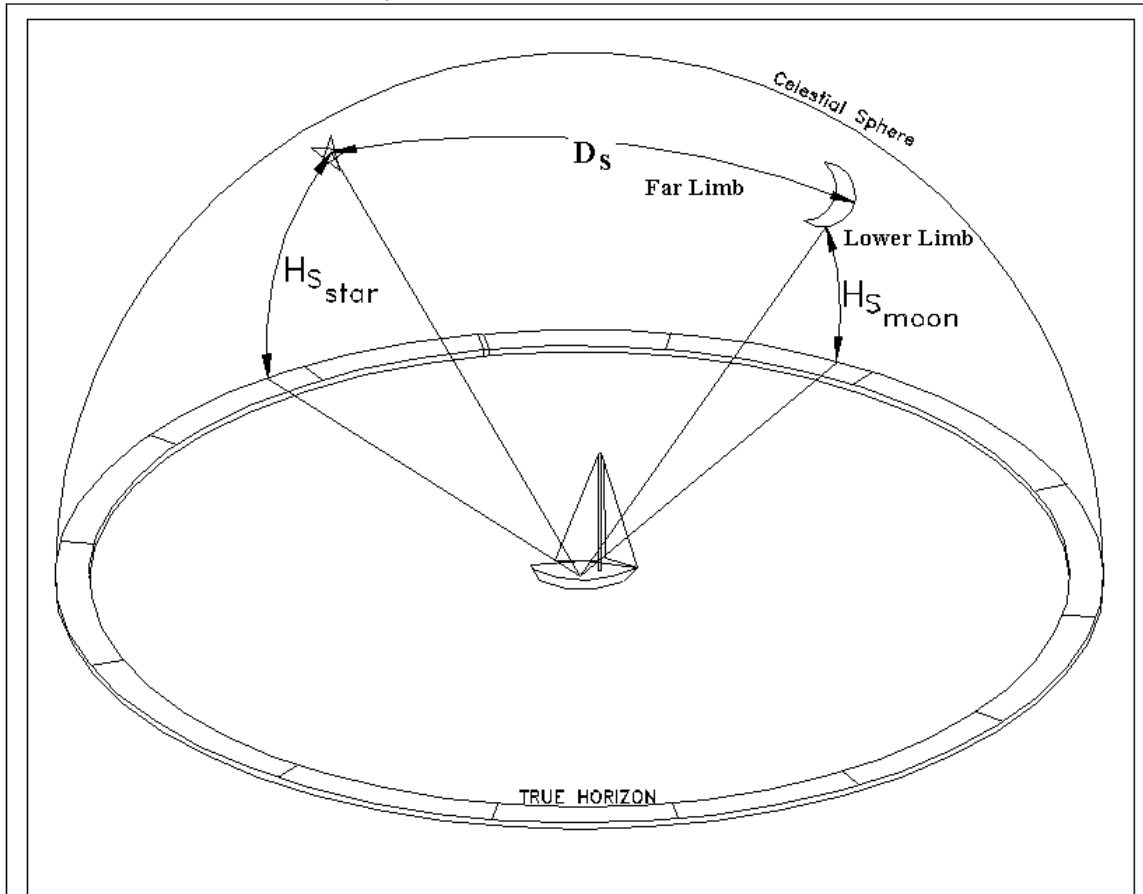
-------------------------------- Clearing the distance from a lunar observation --------------------------------------

IC   **Index Error Correction, degrees**

Change all angle data into decimal degrees

h_{eye} **Eye Height above seal level, meters**

Hs_{star} **Measured Altitude of star or planet with Sextant Scale, deg**

Hs_{moon} **Measured Altitude of the Moon with Sextant Scale, deg**

Ds   **Measured arc distance from Lunar limb to star or planet center with Sextant Scale, deg**

UT_{s} **Your imperfect clock time noted at the observation of Ds, convert to decimal hours**

sgn_{limbH} **When measuring altitude, lower limb is +1. upper limb is -1**

sgn_{limbD} **When measuring arc distance, near limb is +1. far limb is -1**

HP   **Horizontal Parallax HP, from the nautical almanac, degrees**

SD_{moon} **=**   **0.2724·HP**   **Lunar semi-diameter, degrees**

Dip Correction

Heye	DipC
0.7m	-1.5'
1.3m	-2'
2.0m	-2.5'
2.9m	-3'
3.9m	-3.5'
5.1m	-4'
6.4m	-4.5'
8.1m	-5.0'
9.8m	-5.5'
11.6m	-6.0'

Refraction correction

Ha	R
10 deg	-5'
13 deg	-4'
15 deg	-3.5'
17 deg	-3'
20 deg	-2.5'
24 deg	-2'
31 deg	-1.5'
41deg	-1'
59 deg	-0.5'
85 deg	0

From this table, determine the refraction correction for the star and the moon

Record values for:

R_{star}

Convert the dip and refraction corrections to decimal degrees !!

R_{moon}

$$Ha_{star} = Hs_{star} + IC + DipC \qquad \text{Apparent Altitude of star or planet corrected for dip and sextant errors}$$

$$Ho_{star} = Ha_{star} + R_{star} \qquad \text{True observed altitude of star corrected for refraction}$$

Apparent Altitude of moon corrected for dip, semidiameter and sextant errors

$$Ha_{moon} = Hs_{moon} + IC + DipC + sgn_{limbH} \cdot SD_{moon} \cdot \left(1 + \frac{\sin(Ha_{moon})}{60.5}\right)$$

$$PA = HP \cdot \cos(Ha_{moon}) \cdot \left(1 - \frac{\sin(Lat)^2}{300}\right) \qquad \text{Parallax in altitude, degrees}$$

$$Ho_{moon} = Ha_{moon} + R_{moon} + PA \qquad \text{True observed altitude of moon correcte for refraction and parallax}$$

Apparent distance from observer point of view before refraction corrections to the arc measurement, degrees

$$Da = Ds + IC + sgn_{limbH} \cdot SD_{moon} \cdot \left(1 + \frac{\sin(Ha_{moon})}{60.5}\right)$$

$$C_{ratio} = \left[\frac{\left(\cos(Ho_{star}) \cdot \cos(Ho_{moon})\right)}{\left(\cos(Ha_{star}) \cdot \cos(Ha_{moon})\right)}\right] \qquad \text{Ratio of cosine values}$$

Arc distance from the Earth's center point of view, no more corrections, degrees

$$D_{cleared} = arcCos\left[\left(\cos(Da) + \cos(Ha_{moon} + Ha_{star})\right) \cdot C_{ratio} - \cos(Ho_{moon} + Ho_{star})\right]$$

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Next we get tabular values for GHA and declination for the moon and the star in the UT hour we think we are in. We will designate that hour as "UT1".

$GHA1_{moon}$ $DEC1_{moon}$ $GHA1_{star}$ $DEC1_{star}$

Calculate the arc distance at UT1, no parallax, or refraction (geocentric)

$$D_1 = arcCos\left(Cos\left(GHA1_{moon} - GHA1_{star}\right) \cdot Cos\left(DEC1_{moon}\right) \cdot Cos\left(DEC1_{star}\right) + Sin\left(DEC1_{moon}\right) \cdot Sin\left(DEC1_{star}\right)\right)$$

Next we get tabular values for GHA and declination for the moon and the star at the next UT hour from UT1. We will designate that hour as "UT2". (UT2 = UT1 + 1.0)

$GHA2_{moon}$ $DEC2_{moon}$ $GHA2_{star}$ $DEC2_{star}$

Calculate the arc distance at UT2, no parallax, or refraction (geocentric)

$$D_2 = arcCos\left(Cos\left(GHA2_{moon} - GHA2_{star}\right) \cdot Cos\left(DEC2_{moon}\right) \cdot Cos\left(DEC2_{star}\right) + Sin\left(DEC2_{moon}\right) \cdot Sin\left(DEC2_{star}\right)\right)$$

The fraction of time in decimal hours before (-) or ahead (+) of UT1 hour is   :

$$ATIME = \frac{\left(D_{cleared} - D_1\right)}{D_2 - D_1}$$ Decimal hours, this value can be positive or negative

$$UT_{observation} = UT1 + ATIME$$ Best estimate of the time of observation when the arc distance Ds was measured, decimal hours

$$TIME_{error} = UT_s - UT_{observation}$$ Clock error, decimal hours. Positive means clock is fast, negative clock is slow

NO ITERATION REQUIRED   Convert the decimal hour to minutes by multiplying by 60 For example, TIMEerror = - 0.14765 is: 60 x - 0.14765 = 8.859 minutes slow = 8 minutes 52 seconds slow

The 2nd method with one measurement (the arc distance) will calculate the altitudes based on the assumed position and imperfect chronometer reading. With the new corrected time, you can also correct the assumed longitude, and iterate once or twice more with the better time and longitude results. The basic assumption is that the latitude is more accurate.

Derivation of the equations used in clearing the lunar distance

Law of Sines: $\sin(a)/\sin(A) = \sin(b)/\sin(B) = \sin(c)/\sin(C)$

Law of Cosines: $\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(A)$

(these laws are for spherical trigonometry; there are similar ones for plane trig)

Useful identities:
$\sin(\alpha) = \cos(90° - \alpha)$
$\cos(\alpha) = \sin(90° - \alpha)$

Law of Cosines in terms of co-angles:
$\sin(90-a) = \sin(90-b) \cdot \sin(90-c) + \cos(90-b) \cdot \cos(90-c) \cdot \cos(A)$
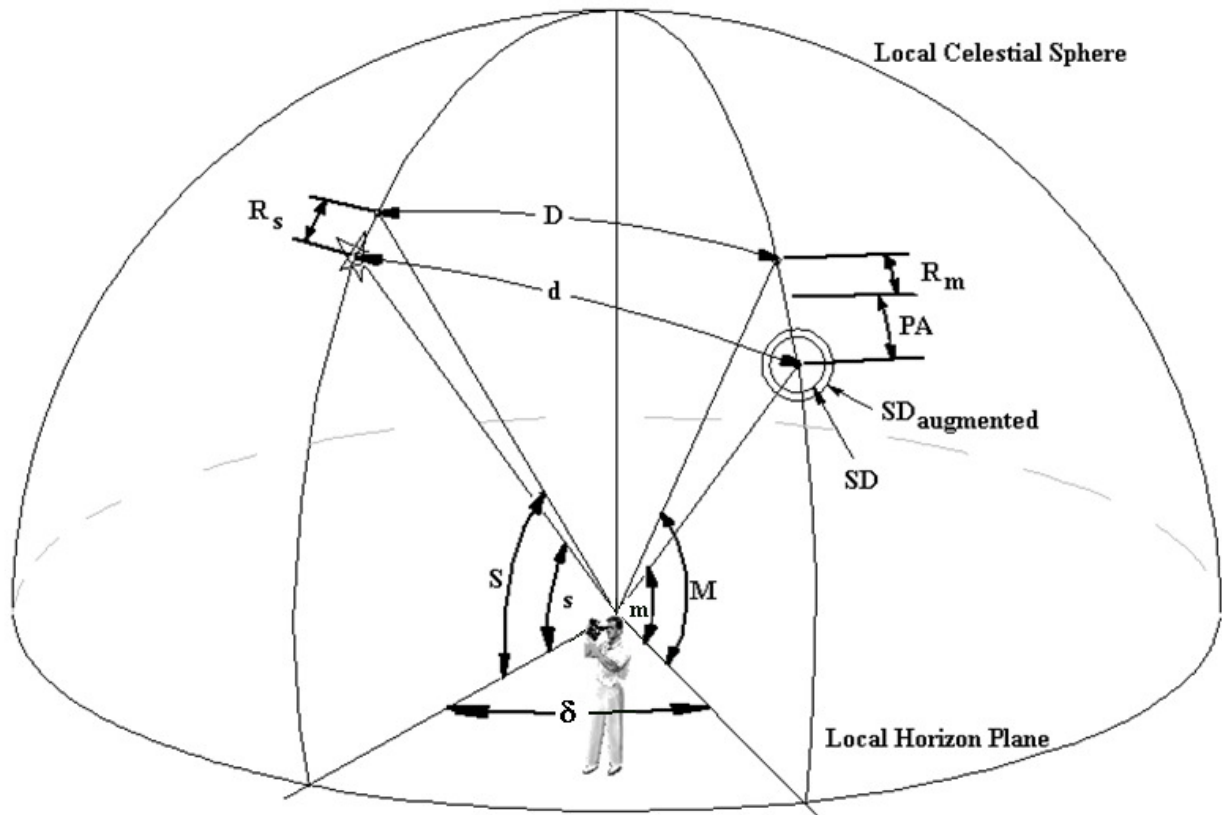
$$S = s + R_S$$
$$M = m + R_m + PA$$
$$d = d_{sextant} + IC + SDaug$$

$$s = Ha_{star} \qquad S = Ho_{star} \qquad m = Ha_{moon} \qquad M = Ho_{moon}$$

$$\cos(d) = \sin(s) \cdot \sin(m) + \cos(s) \cdot \cos(m) \cdot \cos(\delta)$$
$$\cos(D) = \sin(S) \cdot \sin(M) + \cos(S) \cdot \cos(M) \cdot \cos(\delta)$$

Rearrange $\cos(d)$: $\cos(\delta) = \dfrac{\cos(d) - \sin(s) \cdot \sin(m)}{\cos(s) \cdot \cos(m)}$

Now substitute into $\cos(D)$:
$$\cos(D) = \sin(S) \cdot \sin(M) + \cos(S) \cdot \cos(M) \cdot \left[\frac{\cos(d) - \sin(s) \cdot \sin(m)}{\cos(s) \cdot \cos(m)} \right]$$

Rearrange:
$$\cos(D) = \sin(S) \cdot \sin(M) + \left(\cos(d) - \sin(s) \cdot \sin(m) \right) \cdot \left[\frac{\cos(S) \cdot \cos(M)}{\cos(s) \cdot \cos(m)} \right]$$

But using the trig identity:
$$\sin(a) \cdot \sin(b) = -\cos(a + b)$$

And substituting this definition:
$$C_{ratio} = \left[\frac{\cos(S) \cdot \cos(M)}{\cos(s) \cdot \cos(m)} \right]$$

$$\cos(D) = \left(\cos(d) + \cos(s + m) \right) \cdot C_{ratio} - \cos(S + M)$$

$$D = arcCos \left[\left(\cos(d) + \cos(s + m) \right) \cdot C_{ratio} - \cos(S + M) \right]$$

The key to the problem is to realize that the parallax and refraction corrections only change the altitudes, not the 'wedge angle' δ. When the corrections are applied, and using the law of cosines, the equation with s, m can be related to S, M with the common angle δ. Even though refraction corrections are negative, the picture is drawn in the positive direction to establish a consistent sign convention.

Chapter 13 *Coastal Navigation Using the Sextant*

Early in this book a surveyor's technique was mentioned, and it is useful in costal navigation where the relative angle between the observer and 3 identified costal objects are measured. This is the *3-arm protractor* technique (3-point resection in surveyor's tech-speak), and will be described in detail.
A wonderful property of a simple circular arc with 2 end points is that a line drawn from one end point to anywhere on the arc back to the other end point, is the same angle as any other line similarly drawn to another point on the arc.

If you are an observer measuring the relative angle between two known objects on the map, there will exist one unique circle of position where anyone on that arc will measure the same angle between the two known coastal objects.
Include another observation for a third coastal object and make a second angle measurement.   Take for example points A and B on the map (maybe they are water towers or prominent points). An observer measures the relative angle 'a' between them using the sextant held sideways. Then the observer measures an angle 'b' between points B and C (or A and C). The navigator then constructs the two arcs on the map, and where they cross is the position fix. For any arc, if D is the distance between A and B, then the circle's radius R:
R = 0.25 · D · [tan(a/2) + 1 / tan(a/2)]
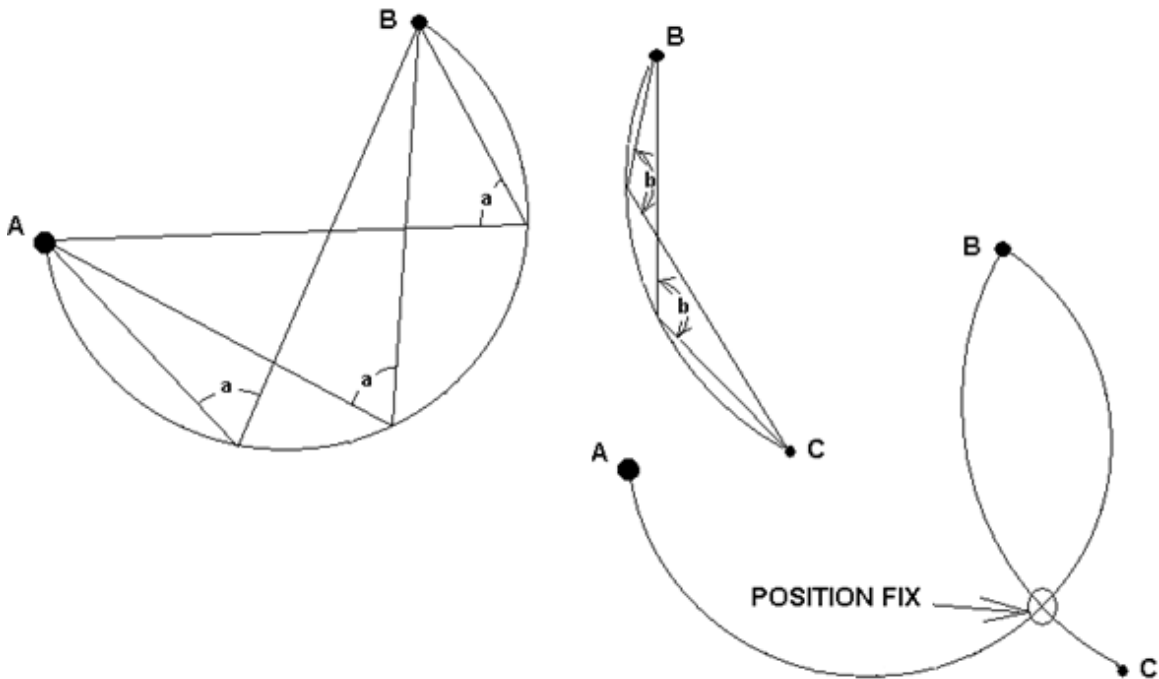
Constructing the arcs by graphical means

Step 1 Draw baseline from point A to point B

Step 2 Draw perpendicular bisector between points A and B
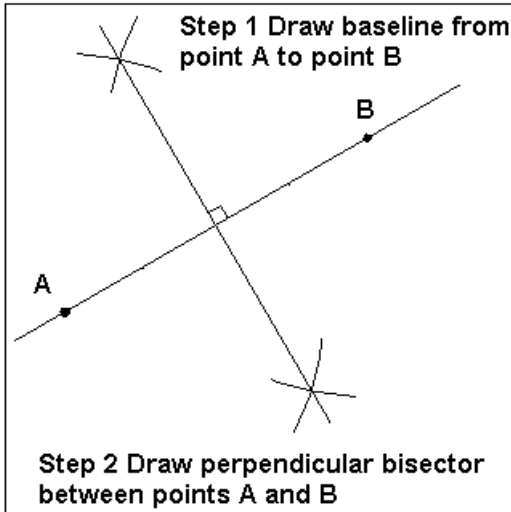
The first step is to draw the baseline between points A and B. Recalling how to draw perpendicular bisectors from middle school geometry using a bow compass, do so for the baseline.

Step 3 With protractor, draw line to bisector with angle $90 - \frac{a}{2}$ from baseline
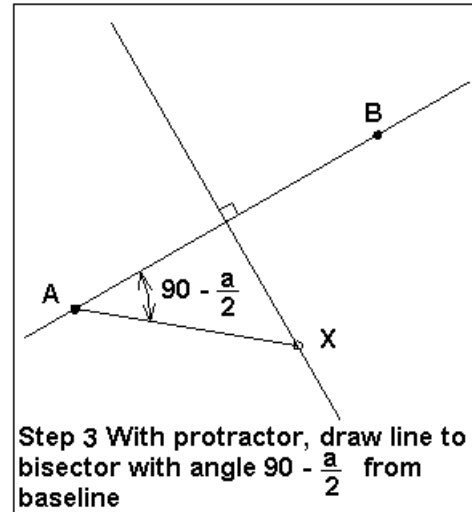
After the bisector is constructed, use a protractor and measure an angle away from the baseline of (90-a/2) from point 'A', if the measured angle with the sextant was 'a' degrees. Where it intersects the bisector, call this point 'X'.

Step 4 Draw perpendicular bisector between points A and X. Carry it to the 1st bisector
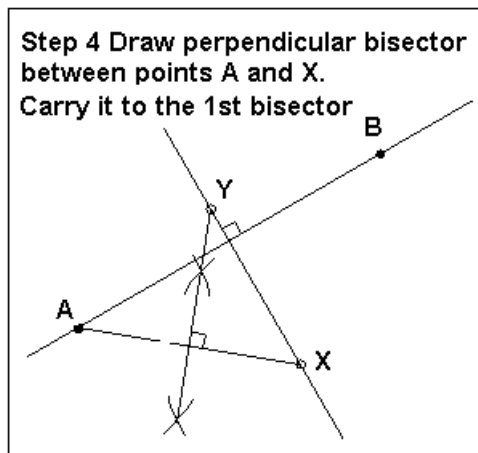
Then draw another perpendicular to split the line A-X, carrying this line until it intersects the first bisector. Call this point 'Y'. It represents the center point of the circle of position.

Step 5 Draw arc with Y as the center, including points A,X and B
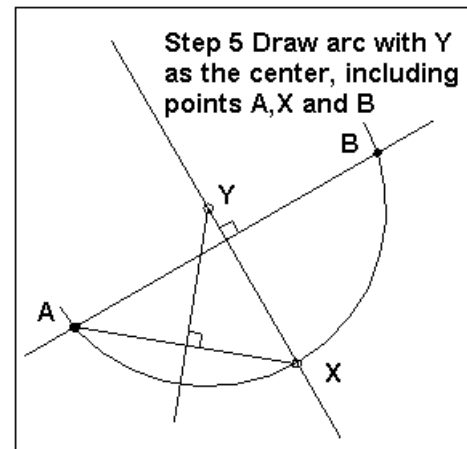
Finally, using point Y as the center, use the bow compass to draw an arc by setting the radius to include either points A, B, or X. This is the circle of position. Repeat steps for drawing the circle of position for points B and C with included angle 'b'. Voila!

Appendix 1 *Generalized Sight Reduction and Intercept Work Sheet*

CELESTIAL NAVIGATION SIGHT REDUCTION EQUATIONS

OBSERVED DATA

True Course **C**	degrees	Last fix Latitude >>	**Lat$_o$**	dec. deg.	
Speed **V**	knots	Last fix Longitude >>	**Lon$_o$**	dec. deg.	
Eye Height **H$_{eye}$**	meters	UT of Last Fix >>	**Time$_o$**	dec. hours	
Temperature >>	deg C	UT Time of this Fix >>	**Time$_{fix}$**	dec. hours	if yesterday, subtract 24
Pressure >>	mbars				

sign$_{limb}$ >> — Which limb, upper or lower — lower limb. = +1 — upper limb. = -1

UT **(H:M:S)** >> — Universal Time of observation

Time = Hour + (Minute + Second / 60) / 60 — Observation time converted to decimal, from hour, minute, second

Hs >> — Measured sextant altitude

I + IC >> — Index and instrument correction

PC >> — Personal correction

Corr^{DIP} = - 0.0293 x SquareRoot(H$_{eye}$) — Dip correction, decimal degrees

Ha = Hs + (IC + I + PC) + Corr^{DIP} — Apparent altitude

SD >> — Semi-diameter — **SD$_{moon}$ = 0.2724 x HP$_{moon}$** — Calculated lunar semi-diameter

HP >> — Horizontal parallax

$$R = \frac{-0.0167}{\tan\left(Ha + \dfrac{7.31}{Ha + 4.4}\right)} \times \left[0.28 \times \frac{Pressure}{(Temperature + 273)}\right]$$

— Refraction correction, decimal degrees

PA = HP x Cos(Ha) — Parallax-in-Altitude

Corr^{ALT} = R + SD x sign$_{limb}$ + PA — Total altitude corrections, or just get directly from the almanac and convert to dec.deg.

Ho = Ha + Corr^{ALT} — Observed altitude, all corrections accounted for

ESTIMATED POSITION

ΔTime = (Time$_{fix}$ - Time$_o$) — Time interval from last fix to present fix

D = V x ΔTime — Distance traveled from last fix to present fix

ΔLat = (D/60) x Cos(C) — Change of latitude from last fix, decimal degrees

ΔLon = (D/60) x Sin(C) / Cos(Lat$_o$ + ΔLat2) — Change of longitude from last fix, decimal degrees

DR Latitude = (Lat$_o$ + ΔLat) — Estimated latitude at time of fix, decimal degrees

DR Longitude = (Lon$_o$ + ΔLon) — Estimated longitude at time of fix, decimal degrees

Roffset = V x (Time$_{fix}$ -Time) — Offset distance for a running fix correction, n.m.

NAUTICAL ALMANAC DATA, GHA and Declination

Determine GHA at the specific Universal Time

	GHA at the hour, decimal degrees	GHA nominal hourly rates		
			Sun, planets	Aries	Moon
GHA_{HR} >>			15.0000	15.0411	14.3167
variance v >>	arc-minutes/hr				
GHA rate >>	<<<<<<<<<<<<<<				
SHA >>	if sun or planets, leave blank				

Δt = Time - Integer(Time)   Hour fraction, mantissa of decimal observation time: **M/60 + S/3600**

$Corr^{GHA}$ = rate x Δt GHA increment due to minutes and seconds

$Corr^v$ = v x Δt/60   Applicable variance correction, decimal degrees

GHA = SHA + GHA_{HR} + $Corr^{GHA}$ + $Corr^v$ GHA at the Universal Time of observation

Determine Declination at the specific Universal Time

	Declination at the hour, dec. deg.	N +, S -
DEC_{HR} >>		
rate d >>	arc-minutes/hr	if the rate is northwards + if the rate is southwards -

$Corr^d$ = d x Δt/60   decimal degrees

DEC = DEC_{HR} + $Corr^d$ Declination at the Universal Time of observation, degrees

Using The GHA and Declination Values, Determine Calculated Altitude Hc

	dec.deg.	Assumed Latitude
Lat_A >>		
Lon_A >>	dec.deg.	Assumed Longitude

LHA = GHA + Lon_A   Local hour angle, post meridian if 0 < LHA < 180, and pre meridian otherwise (which includes - LHA)

$$Hc = arcSin\left(Sin(DEC)\cdot Sin(Lat_A) + Cos(Lat_A)\cdot Cos(DEC)\cdot Cos(LHA)\right)$$ Calculated Altitude, decimal degrees

$$Zo = arcCos\left(\frac{Sin(DEC) - Sin(Lat_A)\cdot Sin(Hc)}{Cos(Lat_A)\cdot Cos(Hc)}\right)$$ Uncorrected azimuth angle, decimal degrees

Azimuth = Zo if LHA is pre-meridian   Corrected Azimuth angle from true north, decimal degrees
Azimuth = 360- Zo if LHA is post-meridian (0 < LHA < 180)

Doffset = 60 x (Ho - Hc)   Offset distance to the intercept LOP, nm (+ move towards, - move away)

Appendix 2   *Making Your Very Own Octant*

Frames for octants can be made from just about any clear wood. In the case of the author's octant, it was made from ¾ inch thick clear maple, and epoxied to form the fine-boned frame shown here. The mirrors are indexed to their position using 3 brads, 2 along the bottom forming a horizontal line, and the third brad along the side to index side-to-side motion. Brass shim stock cut into rectangles and formed over a round pencil produced the U-shaped mirror retaining springs. One-inch long #4-40 screws and nuts are used to make a 3-point adjustable platform for mirror alignment

**Farley Octant
12" radius**
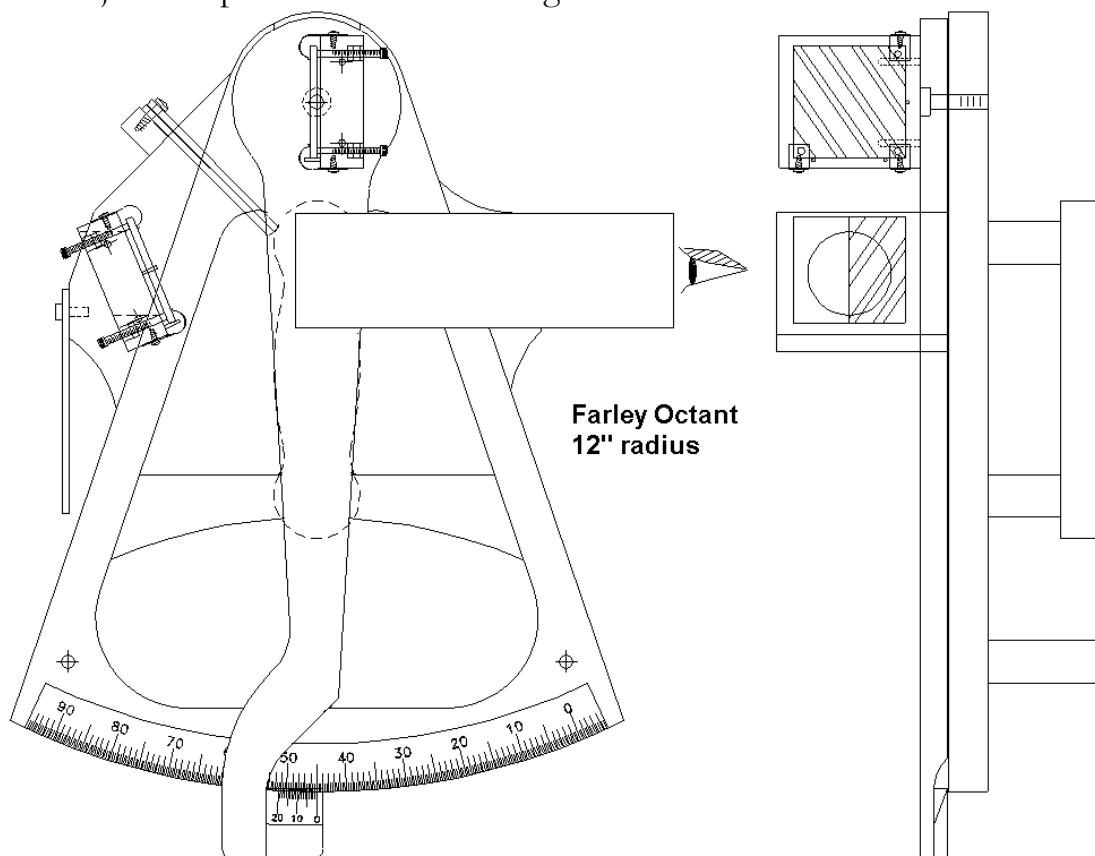
The arc degree scale and Vernier scale were drawn in a 2-D computer aided design program and printed out at 1:1 scale. The laser and bubble jet printers of today are amazingly accurately. The Vernier scale should not go edge to edge with the degree scale, but rather overlap it on a tapered ramp. This means that you do not need to sand the wood edge perfectly arc-shaped, so only the degree scale needs to be placed with accuracy. The Vernier scale is moved radially in and out until it lines up perfectly with the degree scale, only then is it glued to the index arm.

Mirrors

Surprisingly good mirrors can be found in craft stores, 2"x2" for about 25¢ each. Terrible mirrors can be had at the dollar store out of compacts. The quality can be surmised by tilting the mirror until you are seeing a small glancing reflection of something. Ripples (slope errors) will be quite evident at these high reflection angles. The ripples may be just in one direction, and so the mirror can be oriented on the sextant to minimize altitude distortions. The next best is to order a second surface mirror (50mm square) from an optics house such as Edmunds Scientifics for about $4. In their specialty house, you can order first surface mirrors for maybe $20. The second surface mirrors are good enough for a homemade (and professional) sextant. Removing the aluminized surface for the horizon mirror requires patience, and is best accomplished with a fixture to hold the mirror and a guide for the tool. The back has a protective coating that must be removed to get to the reflective material. For a tool, I use a very well sharpened/honed 1" wide wood chisel. The edges should be slightly rounded so as not to dig in. Under no circumstances should you use a scotch-brite pad to remove the silvering, as it will scratch glass. The silvering can best be removed with a metal polisher such as Brasso, using a soft cloth.

Shades

Shades for the sky and horizon filters can be made from welder's mask replacement filter plates, available at welding supply houses for about $1.65. They cut out 99.9% of harmful UV and infrared heat as well as act as neutral density filters to reduce the over-all amount of visible light. The welding shades are numbered 1 thru 16, 1 being the lightest and 16 the darkest. Shades can be additive, that is a #5 shade plus a #6 shade is equivalent to a #11 shade. A #4 shade allows about 13% visible transmission, while a #5 allows around 5%. Shades equivalent to a commercial sextant (by unscientific methods) is approximately 14, 10, 4 for the sky filters and 8, 4 for the horizon filters. Most of these welder's shades will turn the Sun green. Replacement shade filter plates typically can be found for 4 thru 14. Use a 5, a 10, and a 14, which would seem to cover all viewing situations without having to double-up on filters (the glass is not perfect, and more than one filter will distort the Sun's image slightly). A 4 and 6 for the horizon will give 4, 6, and 10. The problem of contrast arises, a green sun disk on a green horizon. But safety of your eyes is paramount, no sense of increasing chances of cataracts due to ultraviolet overexposure. Buy the plates in a 2 by 4.25 inch size, and cut them in half to make 2 squares. Now glass cutting these thick plates is no laughing matter. I have found that if you score lines with a handheld glass cutter on the front <u>and</u> back (and edges too) so that the lines are right over each other, you stand a much better chance of a successful cut. This will require practice…

Springs

Torsion springs to hold the mirrors in place can be easily made by wrapping thin (0.015") music wire around larger diameter music wire or brad nails. Leaf type springs can be cut out from 0.010" brass sheet stock or tin can lids, and wrapped around a pencil to get a 'U' shape.

Sighting telescope

A simple Galilean telescope can be made with a convex lens for the objective lens, and a concave lens for the eyepiece. The image will be upright, and the magnification need not be greater than 3. The convex lens has a positive focal length (FL1), while the concave lens has a negative focal length (FL2). The spacing 'S' between the lenses should be FL1+FL2, and the magnification 'M' is -FL1/FL2. For example, if the objective lens has a focal length of 300mm and the eyepiece lens has a focal length of -150mm, then:

Spacing S = FL1 + FL2 = 300 + (-150) = 150mm

Magnification M = - (FL1/FL2) = -(300 / (-150)) = 2

Edmunds Scientifics sells 38mm diameter lenses for about $3 to $4 each. The tubes can be made with a square cross section using basswood or thin hobby plywood.

LENS HOLDER FRAME   1st BAFFLE   2nd BAFFLE   EYEPIECE HOLDER

OBJECTIVE LENS   EYEPIECE LENS

BOX TELESCOPE
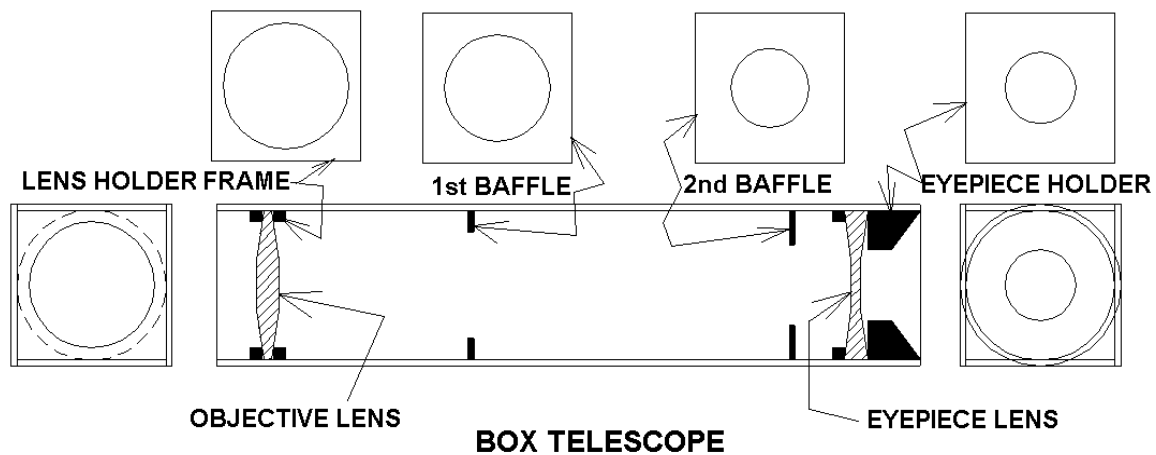
Paint the insides of the tube flat black. The baffles are used to keep stray light from glaring up the insides of the tube, which then reflect into the eyepiece. These baffles effectively trap the unwanted light. Generally speaking, the more baffles, the better the image contrast.

Photos of the Octant

Making of the telescope   Horizon mirror and mount
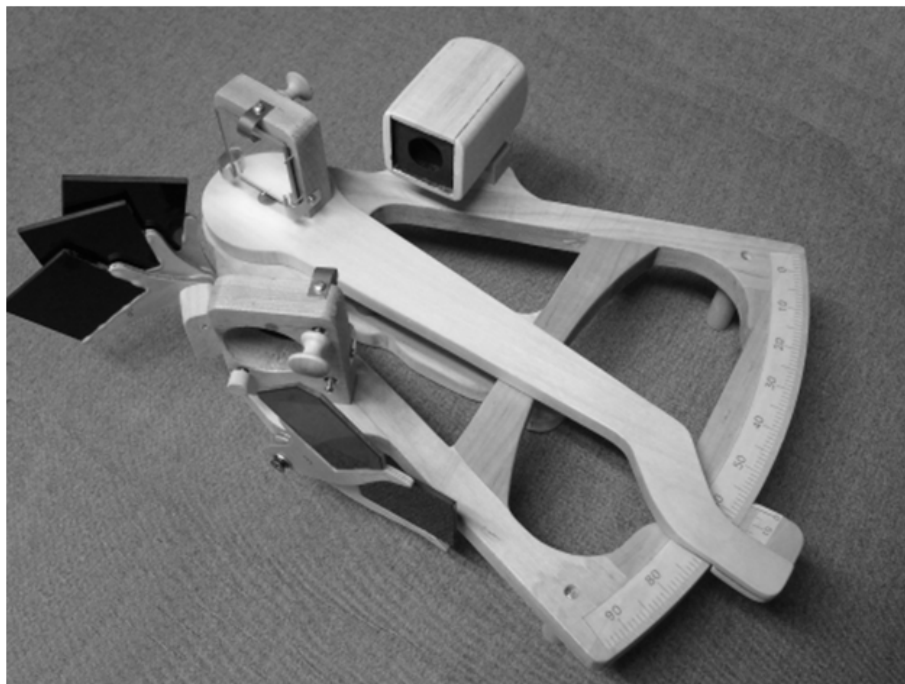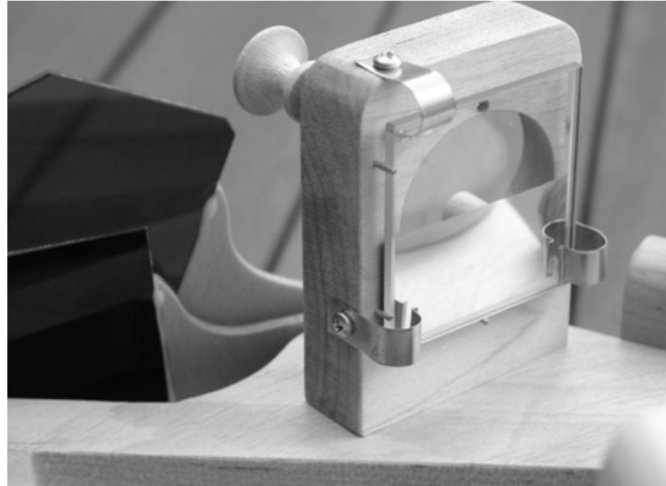
The completed Octant

Appendix 3 *On-Line Resources for Celestial Navigation*

Star Path navigational school
http://www.starpath.com/resources/cellinks.htm

Celestaire
http://celestaire.com/catalog/

On-line nautical almanac
http://www.tecepe.com.br/scripts/AlmanacPagesISAPI.isa

US Naval Observatory
http://aa.usno.navy.mil/data/docs/celnavtable.html

Celestial navigation net- good all around source
http://www.celestialnavigation.net/index.html

A short guide to celestial navigation and freeware
http://home.t-online.de/home/h.umland/index.htm

Official UTC time
http://nist.time.gov/timezone.cgi?UTC/s/0/java

International Earth Rotation Service, gives delta T for Ephemeris Time
http://maia.usno.navy.mil/

Edmunds Scientifics, supplier of mirrors and lenses
http://www.scientificsonline.com/

Edmund Optics, higher grade of optics
http://www.edmundoptics.com/catalog/

American Science and Surplus, with all sorts of spare optical stuff
http://www.sciplus.com/

My web site: www.teacupnavigation.net

Appendix 4   **S-Tables**

Celestial Navigation via the S-Tables
And Ageton's Method

By Teacup Navigation

Rodger E. Farley

Contents

Introduction

Ageton devised a method of dividing the navigational triangle into 2 simpler right-angle triangles to solve. Using the Ageton equations for calculated altitude and meridian angle, a step by step solution to the problem can be implemented using tables and simple addition/subtraction. Ageton reformulated his equations with secant and co-secant functions, but I have retained the original with sine, cosine. The tables represent the log (base 10) of sine and cosine of angles, and then multiplied by -100 for ease.

These are the basic Ageton's equations:

$$R := asin(sin(LHA) \cdot cos(DEC))$$

$$K := asin\left(\frac{sin(DEC)}{cos(R)}\right)$$

$$H_c := asin(cos(R) \cdot cos(K - Lat))$$

$$Z_o := asin\left(\frac{sin(R)}{cos(H_c)}\right) \quad \text{Meridian angle}$$

if $|K| > |Lat|$, then $K_{type} = 1$, else it equals 0

if sign(Lat) x sign(DEC) x $K_{type} = 1$, then Zo is correct, else the corrected Zo equals 180-Zo
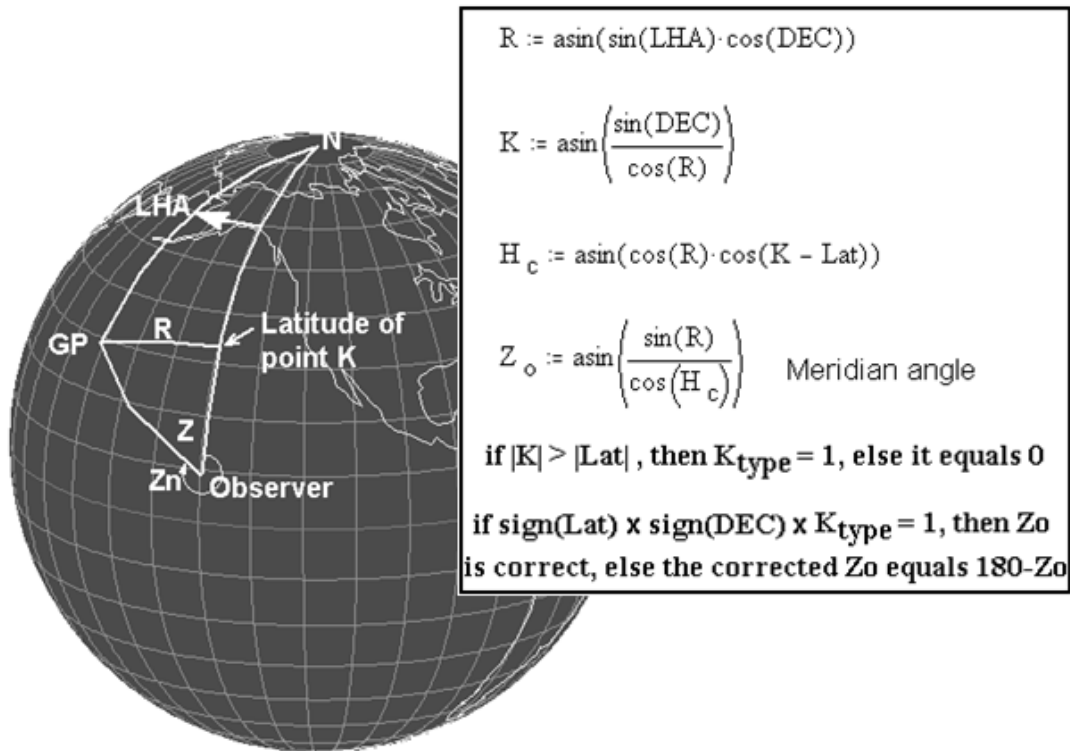
Corrected meridian angle then needs to be converted to true azimuth angle, Zn.

Determining Local Hour Angle LHA

North Lat is +, South Lat is -   East Lon is +, West Lon is -

The Nautical Almanac hourly tabular values of Greenwich Hour Angle (**GHA$_{hour}$**)is corrected for minutes and seconds increments (**Corr ^{GHA}**) plus any hourly variances **v** (**Corr ^{V}** for the minutes). For stars **GHA = SHA + GHA^{Aries}**

Hourly values of declination (**DEC$_{hour}$**) are corrected for minutes with hourly rate **d** (**Corr ^{d}** for the minutes). Beware of using the correct sign [+/-] in rate **d**. If the declinations are moving in a northerly direction, then the sign of **d** is positive (+) even if the declination is still southern. If **d** is moving in a southerly direction (Dec becoming more southerly in the next hour) then d is negative (-), even if the declination is still northern.

GHA = GHA$_{hour}$ + Corr ^{GHA} + Corr ^{V}
DEC = DEC$_{hour}$ + Corr ^{d}
If **DEC** is N then it is +, if S, then **DEC** is -

Using the correct sign (+/-) for longitude **LON**:
LHA = GHA + LON
If LHA > 360, then subtract 360.

LHA is divided into two camps: post-meridian passage and pre-meridian passage. Post meridian angles range from zero to 180 degrees (0<LHA<180), and pre meridian angles range from 180 to 360 (180<LHA<360). For a computer program, pre-meridian LHAs need to be converted to a negative angle (-180<LHA<0) by subtracting 360 from a pre-meridian angle(for example 276 is equivalent to - 84) . The tables and rules can handle both forms if you are careful!
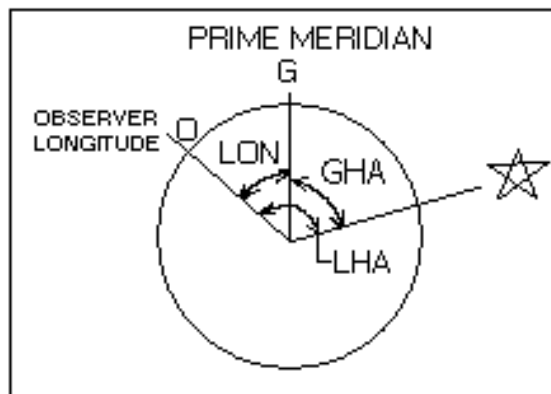
Individual Steps

In the following algorithm, the arrows with "tables" mean go to the S-tables and look at the numbers in the columns. In each column there are 2 numbers side-by-side. If the number sought is an S number, then the one in the same column next to it is a C number. If you have an arrow with "tables/angle", it means locate the given S number, and note the corresponding angle. There are 4 choices of angle, but you will select from the 2 bold numbers, and the given rules will make it clear which of the two to use. Also note that calculated altitude Hc is always less than 90 deg.

1) With |LHA| in tables → S1
2) With |DEC| in tables → S2, C2
3) S3 = S1 + C2, in tables find corresponding C3
4) S4 = S2 - C3, in tables corresponding angle = K
   $$K > 90 \text{ if LHA between 90 and 270}$$
5) K' = |K - LAT x sign(DEC) |, in tables → C5
6) S6 = C5 + C3, in tables → C6
7) Also, in tables corresponding angle of S6 = Hc
8) S7 = |S3 – C6|, in tables corresponding angle = Zo
   Choose the bold-faced angle greater than 90, unless LAT and DEC same name AND |K| > |LAT|, in which case choose the angle less than 90.

Notes:
A variable in straight brackets, for example |LHA|, means to take the absolute value of LHA. In that if it is negative, then make it positive. If LHA = -12, then |LHA| = 12. A greater-than sign is > and a less-than sign is < (K>90 means K greater than 90). When there is a variable prefixed with sign(variable), it means determine the sign and assign it a +1 value if it's positive, or a -1 value if the sign of the number is negative.

Converting Corrected Meridian angle Zo to true azimuth Zn
Name of LAT to select **N** or **S** prefix. For **E** or **W** suffix:
If LHA pre-meridian (-180<LHA<0 or 180<LHA<360) then **E**
If LHA post-meridian (0<LHA<180) then **W**

NW: Zn = 360 - Zo   **NE**: Zn = Zo
SE: Zn = 180 - Zo   **SW**: Zn = Zo + 180

Log Entry

UTC Date

Body and Limb

Approx Azimuth

DR Lat   N / S   DR Lon   W / E

Eye height   meters

Corr^{DIP} =
Corr^{ALT} =
IC =
Total corr =

N LAT is +   E LON is +
S LAT is -   W LON is -

Ha = Hs + IC + Corr^{DIP}
Ha =

Ho = Ho + IC + Corr^{ALT}
Ho =

	deg	arc min
SHA		
GHA hr		
increment		
GHA		

LHA = GHA + Lon

| Lon | | |
| LHA | | |

DEC hr		
increment		minutes
DEC		

Ho =
from work → Hc =
sheet below

Doffset =

Sextant Altitudes   UTC

deg	min	H	M	S

Hs

Graph the sextant altitudes vs. time
Draw a line to average out the random errors
Choose one time and corresponding Hs
from the averaged line. Print values below.

Hs =
UTC =

Time in 5 or 10 second blocks

Sextant angle in 1 arcminute blocks

The offset distance D$_{OFFSET}$ to determine the true line of position is equal to:

D$_{OFFSET}$ = (Ho - Hc), altitudes in minutes of arc. D$_{OFFSET}$ in nautical miles for both cases. If D$_{OFFSET}$ is positive, then parallel offset your assumed line of position in the azimuth direction towards the heavenly object. If negative, then draw it away from the heavenly object.

LHA = + / - [deg | arcmin]

absolute value of |LHA| → tables

S1 [ ][ ]
+ C2
= S3 → tables → C3 [deg | arcmin]

LAT = + / - [deg | arcmin]

DEC = + / - [deg | arcmin]

absolute value of |DEC| → tables

S2 [ ][ ]
- C3
= S4 → tables → angle → K [deg | arcmin] ◁ note

K > 90 if LHA between 90 and 270
(or between -90 and 90 [-90,-170,150,110, etc])

K [deg | arcmin]
+/- |LAT| If LAT and DEC same name then subtract (-)
= → absolute value → K' [deg | arcmin]

C5 [ ][ ]
+ C3
= S6 → tables → tables → angle → Hc [deg | arcmin]

S3 [ ][ ]
- C6
= → absolute value → S7 → tables → angle → Zo

C6 → tables [deg | arcmin]

less than 90 if
☐ LAT and DEC same name
☐ AND |K| > |LAT|

The arrows with "tables" means go to the S-tables and look at the numbers in the columns. In each column there are 2 numbers side-by-side. If the number sought is an S number, then the one in the same column next to it is a C number. If you have an arrow with "tables/angle", it means locate the given S number, and note the corresponding angle. There are 4 choices of angle, and the given rules will make it clear. When in doubt, use the bold-faced numbers. Also, Hc is always less than 90 deg.

N or S prefix: name of LAT determines N or S
E or W suffix:
If LHA pre-meridian (-180<LHA<0 or 180<LHA<360) then **E**
If LHA post-meridian (0<LHA<180) then **W**

NW: Zn = 360 - Zo   NE: Zn = Zn
SE: Zn = 180 - Zo   SW: Zn = Zo + 180

Zo [ ] deg → Zn [ ] deg

111

LHA = + / − [9][2] (deg arcmin) absolute value of |LHA| → tables

S1 [80.408]
+ C2 [0.957]
= S3 [81.364]

C3 [0.518] ← tables

LAT = + / − [40][12] (deg arcmin) absolute value of |DEC| → tables

S2 [68.272]
− C3 [0.518]
= S4 [67.753]

DEC = +/− [11][59] (deg arcmin)

K [12][7.5] (deg arcmin) ← tables angle

K > 90 if LHA between 90 and 270
(or between −90 and 90 [−90,−170,150,110, etc]) ← note

K (deg arcmin) [12][7.5]
− |LAT| [40][12]
= [52][19.7]   (+) / − : If LAT and DEC same name then subtract (−)

K′ [52][19.7] (deg arcmin) ← absolute value

C5 [21.391]
+ C3 [0.518]
= S6 [21.909]   ← tables

Hc [37][9] (deg arcmin) ← tables angle

S3 [81.364]
− C6 [9.851]
= [71.513]   ← tables

C6 [9.851] ← tables

S7 [71.513] ← absolute value

Zo [71.513] ← tables angle

Zo [169] deg

Zn [11] deg

The arrows with "tables" means go to the S-tables and look at the numbers in the columns. In each column there are 2 numbers side-by-side. If the number sought is an S number, then the one in the same column next to it is a C number. If you have an arrow with "tables/angle", it means locate the given S number, and note the corresponding angle. There are 4 choices of angle, and the given rules will make it clear. When in doubt, use the bold-faced numbers. Also, Hc is always less than 90 deg.

N or S prefix: name of LAT determines N or S
E or W suffix:
If LHA pre-meridian (−180<LHA<0 or 180<LHA<360) then **E**
If LHA post-meridian (0<LHA<180) then **W**

NW: Zn = 360 − Zo   NE: Zn = Zn
SE: Zn = 180 − Zo   SW: Zn = Zo + 180

greater than 90
less than 90 if
□ LAT and DEC same name
□ AND |K| > |LAT|

This is an example where the LHA was already figured from the GHA and assumed longitude. The assumed latitude is -40° 12' S. Just work it out line by line to see the genesis of the numbers with the corresponding tabular values.

Using the S-Tables

In this section I will provide examples of how to navigate the tables.
In the tables, these are the equations tabulated to the arc minute:
If x is the angle, then $S(x) = -100*\log(\sin(x))$, and $C(x) = -100*\log(\cos(x))$.
$S(x)$ and $C(x)$ are the S and C values associated with the angle x. Why go to this trouble? Because logarithms change multiplication into addition, and addition is much easier!

If declination **DEC** is 20° 13' S, that means **DEC** = -(20° 13'), and
|**DEC**| = 20° 13'. In the tables look up the bold-faced degree **20** (S-Tables, P5). That heading is on the top of the table which means the arc minutes to use are on the left hand side as indicated by the double arrows. Go down to minute 13 and look across to the column for the 20 deg. That number (46.146) is the S number, and the one next to it in the same column is the C number

←	200°	290°	201°
←	20°	110°	21°
0'	46.595	2.701	44.567
1	46.560	2.706	44.534
2	46.526	2.711	44.501
3	46.491	2.715	44.469
4	46.456	2.720	44.436
5	46.422	2.724	44.403
6	46.387	2.729	44.370
7	46.353	2.734	44.337
8	46.318	2.738	44.305
9	46.284	2.743	44.272
10	46.249	2.748	44.239
11	46.215	2.752	44.207
12	46.181	2.757	44.174
13	46.146	2.762	44.142
14	46.112	2.766	44.109
15	46.078	2.771	44.077

(2.762). If the number you were looking up was 110° 13', then 2.762 would be the S number, and 46.146 would be the C number. Get it?

Using the bottom row of numbers is equally easy. If you are looking up 85° 5', then 0.160 is the S number and 106.699 is the C number. The bottom row of numbers means you use the right-hand minutes.

0.101	106.846	0.159	6
0.102	106.699	0.160	5
0.102	106.552	0.161	4
0.103	106.406	0.162	3
0.104	106.260	0.163	2
0.105	106.115	0.164	1
0.106	105.970	0.166	0'
86°	175°	85°	→
266°	355°	265°	→

Follow the calculation steps from top to bottom, adding or subtracting where indicated. For the values of K, Hc, and Zo you have "go backwards" thru the tables (an S value is given and you have to determine the corresponding angle).

S-Tables, P1

←	180°	270°	181°	271°	182°	272°	183°	273°	184°	274°	
←	0°	90°	1°	91°	2°	92°	3°	93°	4°	94°	
0′	1000.000	0.000	175.815	0.007	145.718	0.026	128.120	0.060	115.642	0.106	60′
1	353.627	0.000	175.097	0.007	145.358	0.027	127.880	0.060	115.461	0.107	59
2	323.524	0.000	174.391	0.007	145.001	0.027	127.641	0.061	115.282	0.108	58
3	305.915	0.000	173.696	0.007	144.646	0.028	127.403	0.062	115.103	0.109	57
4	293.421	0.000	173.012	0.008	144.295	0.028	127.166	0.062	114.925	0.109	56
5	283.730	0.000	172.339	0.008	143.946	0.029	126.931	0.063	114.748	0.110	55
6	275.812	0.000	171.676	0.008	143.600	0.029	126.697	0.064	114.571	0.111	54
7	269.118	0.000	171.023	0.008	143.257	0.030	126.465	0.064	114.395	0.112	53
8	263.318	0.000	170.379	0.008	142.916	0.030	126.233	0.065	114.220	0.113	52
9	258.203	0.000	169.745	0.009	142.579	0.031	126.003	0.066	114.045	0.114	51
10	253.627	0.000	169.121	0.009	142.243	0.031	125.774	0.066	113.872	0.115	50
11	249.488	0.000	168.505	0.009	141.911	0.032	125.546	0.067	113.699	0.116	49
12	245.709	0.000	167.897	0.010	141.581	0.032	125.320	0.068	113.526	0.117	48
13	242.233	0.000	167.298	0.010	141.253	0.033	125.095	0.068	113.355	0.118	47
14	239.015	0.000	166.708	0.010	140.928	0.033	124.870	0.069	113.184	0.119	46
15	236.018	0.000	166.125	0.010	140.605	0.033	124.647	0.070	113.013	0.120	45
16	233.216	0.000	165.550	0.011	140.285	0.034	124.425	0.071	112.844	0.121	44
17	230.583	0.001	164.982	0.011	139.967	0.034	124.205	0.071	112.675	0.121	43
18	228.100	0.001	164.422	0.011	139.651	0.035	123.985	0.072	112.506	0.122	42
19	225.752	0.001	163.869	0.011	139.338	0.036	123.766	0.073	112.339	0.123	41
20	223.525	0.001	163.322	0.012	139.027	0.036	123.549	0.074	112.172	0.124	40
21	221.406	0.001	162.783	0.012	138.718	0.037	123.333	0.074	112.005	0.125	39
22	219.385	0.001	162.250	0.012	138.411	0.037	123.117	0.075	111.839	0.126	38
23	217.455	0.001	161.724	0.013	138.106	0.038	122.903	0.076	111.674	0.127	37
24	215.607	0.001	161.204	0.013	137.804	0.038	122.690	0.077	111.510	0.128	36
25	213.834	0.001	160.690	0.013	137.504	0.039	122.478	0.077	111.346	0.129	35
26	212.131	0.001	160.182	0.014	137.205	0.039	122.267	0.078	111.183	0.130	34
27	210.491	0.001	159.680	0.014	136.909	0.040	122.057	0.079	111.020	0.131	33
28	208.912	0.001	159.184	0.014	136.615	0.040	121.848	0.080	110.858	0.132	32
29	207.388	0.002	158.693	0.015	136.322	0.041	121.640	0.080	110.697	0.133	31
30	205.916	0.002	158.208	0.015	136.032	0.041	121.433	0.081	110.536	0.134	30
31	204.492	0.002	157.728	0.015	135.744	0.042	121.226	0.082	110.375	0.135	29
32	203.113	0.002	157.254	0.016	135.457	0.042	121.021	0.083	110.216	0.136	28
33	201.777	0.002	156.784	0.016	135.173	0.043	120.817	0.083	110.057	0.137	27
34	200.480	0.002	156.320	0.016	134.890	0.044	120.614	0.084	109.898	0.138	26
35	199.221	0.002	155.861	0.017	134.609	0.044	120.412	0.085	109.740	0.139	25
36	197.998	0.002	155.406	0.017	134.330	0.045	120.211	0.086	109.583	0.140	24
37	196.808	0.003	154.956	0.017	134.053	0.045	120.010	0.087	109.426	0.141	23
38	195.650	0.003	154.511	0.018	133.777	0.046	119.811	0.087	109.270	0.142	22
39	194.522	0.003	154.070	0.018	133.503	0.046	119.612	0.088	109.115	0.143	21
40	193.422	0.003	153.634	0.018	133.231	0.047	119.415	0.089	108.960	0.144	20
41	192.350	0.003	153.202	0.019	132.961	0.048	119.218	0.090	108.805	0.145	19
42	191.304	0.003	152.774	0.019	132.692	0.048	119.022	0.091	108.651	0.146	18
43	190.282	0.003	152.350	0.019	132.425	0.049	118.827	0.091	108.498	0.147	17
44	189.283	0.004	151.931	0.020	132.160	0.049	118.633	0.092	108.345	0.148	16
45	188.307	0.004	151.515	0.020	131.896	0.050	118.440	0.093	108.193	0.149	15
46	187.353	0.004	151.104	0.021	131.633	0.051	118.248	0.094	108.041	0.150	14
47	186.419	0.004	150.696	0.021	131.373	0.051	118.056	0.095	107.890	0.152	13
48	185.505	0.004	150.292	0.021	131.114	0.052	117.866	0.096	107.739	0.153	12
49	184.609	0.004	149.892	0.022	130.856	0.052	117.676	0.096	107.589	0.154	11
50	183.732	0.005	149.496	0.022	130.600	0.053	117.487	0.097	107.439	0.155	10
51	182.872	0.005	149.103	0.023	130.346	0.054	117.299	0.098	107.290	0.156	9
52	182.029	0.005	148.713	0.023	130.093	0.054	117.112	0.099	107.141	0.157	8
53	181.202	0.005	148.327	0.023	129.841	0.055	116.925	0.100	106.993	0.158	7
54	180.390	0.005	147.945	0.024	129.591	0.056	116.739	0.101	106.846	0.159	6
55	179.593	0.006	147.566	0.024	129.342	0.056	116.554	0.102	106.699	0.160	5
56	178.811	0.006	147.190	0.025	129.095	0.057	116.370	0.102	106.552	0.161	4
57	178.042	0.006	146.817	0.025	128.849	0.058	116.187	0.103	106.406	0.162	3
58	177.287	0.006	146.448	0.026	128.605	0.058	116.004	0.104	106.260	0.163	2
59	176.544	0.006	146.081	0.026	128.362	0.059	115.823	0.105	106.115	0.164	1
60′	175.815	0.007	145.718	0.026	128.120	0.060	115.642	0.106	105.970	0.166	0′
	179°	89°	178°	88°	177°	87°	176°	86°	175°	85°	→
	359°	269°	358°	268°	357°	267°	356°	266°	355°	265°	→

S-Tables, P2

	185°	275°	186°	276°	187°	277°	188°	278°	189°	279°	
←	5°	95°	6°	96°	7°	97°	8°	98°	9°	99°	
0'	105.970	0.166	98.077	0.239	91.411	0.325	85.645	0.425	80.567	0.538	60'
1	105.826	0.167	97.957	0.240	91.308	0.326	85.555	0.426	80.487	0.540	59
2	105.683	0.168	97.837	0.241	91.205	0.328	85.465	0.428	80.408	0.542	58
3	105.539	0.169	97.717	0.243	91.103	0.330	85.376	0.430	80.328	0.544	57
4	105.397	0.170	97.598	0.244	91.001	0.331	85.286	0.432	80.249	0.546	56
5	105.254	0.171	97.480	0.245	90.899	0.333	85.197	0.434	80.170	0.548	55
6	105.113	0.172	97.361	0.247	90.798	0.334	85.109	0.435	80.091	0.550	54
7	104.971	0.173	97.243	0.248	90.696	0.336	85.020	0.437	80.012	0.552	53
8	104.830	0.175	97.126	0.249	90.595	0.337	84.931	0.439	79.933	0.554	52
9	104.690	0.176	97.008	0.251	90.494	0.339	84.843	0.441	79.855	0.556	51
10	104.550	0.177	96.891	0.252	90.394	0.341	84.755	0.443	79.777	0.558	50
11	104.411	0.178	96.774	0.253	90.294	0.342	84.667	0.444	79.698	0.560	49
12	104.272	0.179	96.658	0.255	90.193	0.344	84.579	0.446	79.620	0.562	48
13	104.133	0.180	96.542	0.256	90.094	0.345	84.492	0.448	79.542	0.564	47
14	103.995	0.181	96.426	0.258	89.994	0.347	84.404	0.450	79.465	0.566	46
15	103.857	0.183	96.310	0.259	89.894	0.349	84.317	0.452	79.387	0.568	45
16	103.720	0.184	96.195	0.260	89.795	0.350	84.230	0.454	79.309	0.571	44
17	103.583	0.185	96.080	0.262	89.696	0.352	84.143	0.455	79.232	0.573	43
18	103.447	0.186	95.966	0.263	89.598	0.353	84.056	0.457	79.155	0.575	42
19	103.311	0.187	95.852	0.264	89.499	0.355	83.970	0.459	79.078	0.577	41
20	103.175	0.188	95.738	0.266	89.401	0.357	83.884	0.461	79.001	0.579	40
21	103.040	0.190	95.624	0.267	89.303	0.358	83.797	0.463	78.924	0.581	39
22	102.905	0.191	95.510	0.269	89.205	0.360	83.712	0.465	78.847	0.583	38
23	102.771	0.192	95.397	0.270	89.107	0.362	83.626	0.467	78.771	0.585	37
24	102.637	0.193	95.285	0.272	89.010	0.363	83.540	0.468	78.695	0.587	36
25	102.504	0.194	95.172	0.273	88.913	0.365	83.455	0.470	78.618	0.589	35
26	102.371	0.196	95.060	0.274	88.816	0.367	83.369	0.472	78.542	0.591	34
27	102.238	0.197	94.948	0.276	88.719	0.368	83.284	0.474	78.466	0.593	33
28	102.106	0.198	94.836	0.277	88.623	0.370	83.199	0.476	78.390	0.596	32
29	101.974	0.199	94.725	0.279	88.526	0.371	83.114	0.478	78.315	0.598	31
30	101.843	0.200	94.614	0.280	88.430	0.373	83.030	0.480	78.239	0.600	30
31	101.712	0.202	94.503	0.282	88.334	0.375	82.945	0.482	78.164	0.602	29
32	101.581	0.203	94.393	0.283	88.239	0.376	82.861	0.483	78.088	0.604	28
33	101.451	0.204	94.283	0.284	88.143	0.378	82.777	0.485	78.013	0.606	27
34	101.321	0.205	94.173	0.286	88.048	0.380	82.693	0.487	77.938	0.608	26
35	101.192	0.207	94.063	0.287	87.953	0.382	82.609	0.489	77.863	0.610	25
36	101.063	0.208	93.954	0.289	87.858	0.383	82.526	0.491	77.789	0.612	24
37	100.934	0.209	93.845	0.290	87.764	0.385	82.442	0.493	77.714	0.615	23
38	100.806	0.210	93.736	0.292	87.669	0.387	82.359	0.495	77.639	0.617	22
39	100.678	0.211	93.628	0.293	87.575	0.388	82.276	0.497	77.565	0.619	21
40	100.550	0.213	93.519	0.295	87.481	0.390	82.193	0.499	77.491	0.621	20
41	100.423	0.214	93.412	0.296	87.388	0.392	82.110	0.501	77.417	0.623	19
42	100.296	0.215	93.304	0.298	87.294	0.393	82.027	0.503	77.343	0.625	18
43	100.170	0.217	93.196	0.299	87.201	0.395	81.945	0.505	77.269	0.628	17
44	100.044	0.218	93.089	0.301	87.108	0.397	81.863	0.506	77.195	0.630	16
45	99.918	0.219	92.982	0.302	87.015	0.399	81.780	0.508	77.122	0.632	15
46	99.793	0.220	92.876	0.304	86.922	0.400	81.698	0.510	77.048	0.634	14
47	99.668	0.222	92.769	0.305	86.829	0.402	81.617	0.512	76.975	0.636	13
48	99.544	0.223	92.663	0.307	86.737	0.404	81.535	0.514	76.902	0.638	12
49	99.420	0.224	92.558	0.308	86.645	0.405	81.453	0.516	76.829	0.641	11
50	99.296	0.225	92.452	0.310	86.553	0.407	81.372	0.518	76.756	0.643	10
51	99.172	0.227	92.347	0.311	86.461	0.409	81.291	0.520	76.683	0.645	9
52	99.049	0.228	92.242	0.313	86.370	0.411	81.210	0.522	76.610	0.647	8
53	98.926	0.229	92.137	0.314	86.278	0.412	81.129	0.524	76.538	0.649	7
54	98.804	0.231	92.032	0.316	86.187	0.414	81.048	0.526	76.465	0.652	6
55	98.682	0.232	91.928	0.317	86.096	0.416	80.967	0.528	76.393	0.654	5
56	98.560	0.233	91.824	0.319	86.006	0.418	80.887	0.530	76.321	0.656	4
57	98.439	0.235	91.720	0.320	85.915	0.419	80.807	0.532	76.249	0.658	3
58	98.318	0.236	91.617	0.322	85.825	0.421	80.727	0.534	76.177	0.660	2
59	98.197	0.237	91.514	0.323	85.734	0.423	80.647	0.536	76.105	0.663	1
60'	98.077	0.239	91.411	0.325	85.645	0.425	80.567	0.538	76.033	0.665	0'
	174°	84°	173°	83°	172°	82°	171°	81°	170°	80°	→
	354°	264°	353°	263°	352°	262°	351°	261°	350°	260°	→

S-Tables, P3

←	190°	280°	191°	281°	192°	282°	193°	283°	194°	284°	
←	10°	100°	11°	101°	12°	102°	13°	103°	14°	104°	
0′	76.033	0.665	71.940	0.805	68.212	0.960	64.791	1.128	61.633	1.310	60′
1	75.961	0.667	71.875	0.808	68.153	0.962	64.737	1.131	61.582	1.313	59
2	75.890	0.669	71.810	0.810	68.093	0.965	64.682	1.133	61.531	1.316	58
3	75.819	0.672	71.746	0.813	68.034	0.968	64.627	1.136	61.481	1.319	57
4	75.747	0.674	71.681	0.815	67.975	0.970	64.573	1.139	61.430	1.322	56
5	75.676	0.676	71.616	0.818	67.916	0.973	64.519	1.142	61.380	1.325	55
6	75.605	0.678	71.552	0.820	67.857	0.976	64.464	1.145	61.330	1.329	54
7	75.534	0.681	71.488	0.823	67.798	0.978	64.410	1.148	61.279	1.332	53
8	75.464	0.683	71.423	0.825	67.739	0.981	64.356	1.151	61.229	1.335	52
9	75.393	0.685	71.359	0.828	67.681	0.984	64.302	1.154	61.179	1.338	51
10	75.323	0.687	71.295	0.830	67.622	0.987	64.248	1.157	61.129	1.341	50
11	75.252	0.690	71.231	0.833	67.563	0.989	64.194	1.160	61.079	1.344	49
12	75.182	0.692	71.167	0.835	67.505	0.992	64.140	1.163	61.029	1.348	48
13	75.112	0.694	71.104	0.838	67.447	0.995	64.086	1.166	60.979	1.351	47
14	75.042	0.696	71.040	0.840	67.388	0.998	64.032	1.169	60.929	1.354	46
15	74.972	0.699	70.976	0.843	67.330	1.000	63.979	1.172	60.879	1.357	45
16	74.902	0.701	70.913	0.845	67.272	1.003	63.925	1.175	60.830	1.360	44
17	74.832	0.703	70.850	0.848	67.214	1.006	63.871	1.178	60.780	1.364	43
18	74.763	0.706	70.786	0.850	67.156	1.009	63.818	1.181	60.731	1.367	42
19	74.693	0.708	70.723	0.853	67.098	1.011	63.764	1.184	60.681	1.370	41
20	74.624	0.710	70.660	0.855	67.040	1.014	63.711	1.187	60.632	1.373	40
21	74.555	0.712	70.597	0.858	66.982	1.017	63.658	1.190	60.582	1.377	39
22	74.486	0.715	70.534	0.860	66.925	1.020	63.605	1.193	60.533	1.380	38
23	74.417	0.717	70.471	0.863	66.867	1.022	63.552	1.196	60.483	1.383	37
24	74.348	0.719	70.409	0.865	66.810	1.025	63.498	1.199	60.434	1.386	36
25	74.279	0.722	70.346	0.868	66.752	1.028	63.445	1.202	60.385	1.390	35
26	74.210	0.724	70.284	0.870	66.695	1.031	63.393	1.205	60.336	1.393	34
27	74.142	0.726	70.221	0.873	66.638	1.033	63.340	1.208	60.287	1.396	33
28	74.073	0.729	70.159	0.876	66.580	1.036	63.287	1.211	60.238	1.399	32
29	74.005	0.731	70.097	0.878	66.523	1.039	63.234	1.214	60.189	1.403	31
30	73.937	0.733	70.035	0.881	66.466	1.042	63.182	1.217	60.140	1.406	30
31	73.869	0.736	69.972	0.883	66.409	1.045	63.129	1.220	60.091	1.409	29
32	73.801	0.738	69.911	0.886	66.353	1.047	63.076	1.223	60.042	1.412	28
33	73.733	0.740	69.849	0.888	66.296	1.050	63.024	1.226	59.994	1.416	27
34	73.665	0.743	69.787	0.891	66.239	1.053	62.972	1.229	59.945	1.419	26
35	73.597	0.745	69.725	0.894	66.182	1.056	62.919	1.232	59.897	1.422	25
36	73.530	0.748	69.664	0.896	66.126	1.059	62.867	1.235	59.848	1.426	24
37	73.462	0.750	69.602	0.899	66.069	1.062	62.815	1.238	59.800	1.429	23
38	73.395	0.752	69.541	0.901	66.013	1.064	62.763	1.241	59.751	1.432	22
39	73.328	0.755	69.479	0.904	65.957	1.067	62.711	1.244	59.703	1.435	21
40	73.261	0.757	69.418	0.907	65.900	1.070	62.659	1.247	59.654	1.439	20
41	73.194	0.759	69.357	0.909	65.844	1.073	62.607	1.250	59.606	1.442	19
42	73.127	0.762	69.296	0.912	65.788	1.076	62.555	1.254	59.558	1.445	18
43	73.060	0.764	69.235	0.914	65.732	1.079	62.503	1.257	59.510	1.449	17
44	72.993	0.767	69.174	0.917	65.676	1.081	62.451	1.260	59.462	1.452	16
45	72.927	0.769	69.113	0.920	65.620	1.084	62.400	1.263	59.414	1.455	15
46	72.860	0.771	69.053	0.922	65.565	1.087	62.348	1.266	59.366	1.459	14
47	72.794	0.774	68.992	0.925	65.509	1.090	62.297	1.269	59.318	1.462	13
48	72.727	0.776	68.932	0.928	65.453	1.093	62.245	1.272	59.270	1.465	12
49	72.661	0.779	68.871	0.930	65.398	1.096	62.194	1.275	59.222	1.469	11
50	72.595	0.781	68.811	0.933	65.342	1.099	62.142	1.278	59.175	1.472	10
51	72.529	0.783	68.751	0.936	65.287	1.102	62.091	1.281	59.127	1.475	9
52	72.463	0.786	68.690	0.938	65.231	1.104	62.040	1.285	59.079	1.479	8
53	72.398	0.788	68.630	0.941	65.176	1.107	61.989	1.288	59.032	1.482	7
54	72.332	0.791	68.570	0.944	65.121	1.110	61.938	1.291	58.984	1.485	6
55	72.266	0.793	68.510	0.946	65.066	1.113	61.887	1.294	58.937	1.489	5
56	72.201	0.796	68.451	0.949	65.011	1.116	61.836	1.297	58.889	1.492	4
57	72.136	0.798	68.391	0.952	64.956	1.119	61.785	1.300	58.842	1.495	3
58	72.070	0.800	68.331	0.954	64.901	1.122	61.734	1.303	58.795	1.499	2
59	72.005	0.803	68.272	0.957	64.846	1.125	61.683	1.306	58.748	1.502	1
60′	71.940	0.805	68.212	0.960	64.791	1.128	61.633	1.310	58.700	1.506	0′
	169°	79°	168°	78°	167°	77°	166°	76°	165°	75°	→
	349°	259°	348°	258°	347°	257°	346°	256°	345°	255°	→

S-Tables, P4

←	195°	285°	196°	286°	197°	287°	198°	288°	199°	289°	
←	**15°**	**105°**	**16°**	**106°**	**17°**	**107°**	**18°**	**108°**	**19°**	**109°**	
0′	58.700	1.506	55.966	1.716	53.407	1.940	51.002	2.179	48.736	2.433	60′
1	58.653	1.509	55.922	1.719	53.365	1.944	50.963	2.183	48.699	2.437	59
2	58.606	1.512	55.878	1.723	53.324	1.948	50.924	2.188	48.663	2.442	58
3	58.559	1.516	55.834	1.727	53.283	1.952	50.885	2.192	48.626	2.446	57
4	58.512	1.519	55.790	1.730	53.242	1.956	50.847	2.196	48.589	2.450	56
5	58.465	1.523	55.747	1.734	53.200	1.960	50.808	2.200	48.553	2.455	55
6	58.419	1.526	55.703	1.738	53.159	1.964	50.769	2.204	48.516	2.459	54
7	58.372	1.529	55.659	1.741	53.118	1.967	50.731	2.208	48.480	2.464	53
8	58.325	1.533	55.615	1.745	53.077	1.971	50.692	2.212	48.443	2.468	52
9	58.278	1.536	55.572	1.749	53.036	1.975	50.653	2.216	48.407	2.472	51
10	58.232	1.540	55.528	1.752	52.995	1.979	50.615	2.221	48.371	2.477	50
11	58.185	1.543	55.485	1.756	52.955	1.983	50.576	2.225	48.334	2.481	49
12	58.139	1.547	55.441	1.760	52.914	1.987	50.538	2.229	48.298	2.485	48
13	58.092	1.550	55.398	1.763	52.873	1.991	50.500	2.233	48.262	2.490	47
14	58.046	1.553	55.354	1.767	52.832	1.995	50.461	2.237	48.226	2.494	46
15	57.999	1.557	55.311	1.771	52.791	1.999	50.423	2.241	48.189	2.499	45
16	57.953	1.560	55.267	1.774	52.751	2.003	50.385	2.246	48.153	2.503	44
17	57.907	1.564	55.224	1.778	52.710	2.007	50.346	2.250	48.117	2.508	43
18	57.861	1.567	55.181	1.782	52.670	2.011	50.308	2.254	48.081	2.512	42
19	57.814	1.571	55.138	1.785	52.629	2.014	50.270	2.258	48.045	2.516	41
20	57.768	1.574	55.095	1.789	52.589	2.018	50.232	2.262	48.009	2.521	40
21	57.722	1.578	55.052	1.793	52.548	2.022	50.194	2.266	47.973	2.525	39
22	57.676	1.581	55.009	1.796	52.508	2.026	50.156	2.271	47.937	2.530	38
23	57.630	1.585	54.966	1.800	52.467	2.030	50.118	2.275	47.901	2.534	37
24	57.584	1.588	54.923	1.804	52.427	2.034	50.080	2.279	47.865	2.539	36
25	57.539	1.591	54.880	1.808	52.387	2.038	50.042	2.283	47.829	2.543	35
26	57.493	1.595	54.837	1.811	52.346	2.042	50.004	2.287	47.793	2.547	34
27	57.447	1.598	54.794	1.815	52.306	2.046	49.966	2.292	47.758	2.552	33
28	57.401	1.602	54.751	1.819	52.266	2.050	49.928	2.296	47.722	2.556	32
29	57.356	1.605	54.709	1.823	52.226	2.054	49.890	2.300	47.686	2.561	31
30	57.310	1.609	54.666	1.826	52.186	2.058	49.852	2.304	47.651	2.565	30
31	57.265	1.612	54.623	1.830	52.146	2.062	49.815	2.309	47.615	2.570	29
32	57.219	1.616	54.581	1.834	52.106	2.066	49.777	2.313	47.579	2.574	28
33	57.174	1.619	54.538	1.838	52.066	2.070	49.739	2.317	47.544	2.579	27
34	57.128	1.623	54.496	1.841	52.026	2.074	49.702	2.321	47.508	2.583	26
35	57.083	1.627	54.453	1.845	51.986	2.078	49.664	2.326	47.473	2.588	25
36	57.038	1.630	54.411	1.849	51.946	2.082	49.627	2.330	47.437	2.592	24
37	56.993	1.634	54.368	1.853	51.906	2.086	49.589	2.334	47.402	2.597	23
38	56.947	1.637	54.326	1.856	51.867	2.090	49.552	2.338	47.366	2.601	22
39	56.902	1.641	54.284	1.860	51.827	2.094	49.514	2.343	47.331	2.606	21
40	56.857	1.644	54.242	1.864	51.787	2.098	49.477	2.347	47.295	2.610	20
41	56.812	1.648	54.199	1.868	51.748	2.102	49.439	2.351	47.260	2.615	19
42	56.767	1.651	54.157	1.871	51.708	2.106	49.402	2.355	47.225	2.619	18
43	56.722	1.655	54.115	1.875	51.668	2.110	49.365	2.360	47.190	2.624	17
44	56.677	1.658	54.073	1.879	51.629	2.114	49.327	2.364	47.154	2.628	16
45	56.633	1.662	54.031	1.883	51.589	2.118	49.290	2.368	47.119	2.633	15
46	56.588	1.666	53.989	1.887	51.550	2.122	49.253	2.372	47.084	2.637	14
47	56.543	1.669	53.947	1.890	51.511	2.126	49.216	2.377	47.049	2.642	13
48	56.498	1.673	53.905	1.894	51.471	2.130	49.179	2.381	47.014	2.647	12
49	56.454	1.676	53.864	1.898	51.432	2.134	49.142	2.385	46.979	2.651	11
50	56.409	1.680	53.822	1.902	51.393	2.139	49.104	2.390	46.944	2.656	10
51	56.365	1.683	53.780	1.906	51.353	2.143	49.067	2.394	46.909	2.660	9
52	56.320	1.687	53.738	1.910	51.314	2.147	49.030	2.398	46.874	2.665	8
53	56.276	1.691	53.697	1.913	51.275	2.151	48.994	2.403	46.839	2.669	7
54	56.231	1.694	53.655	1.917	51.236	2.155	48.957	2.407	46.804	2.674	6
55	56.187	1.698	53.614	1.921	51.197	2.159	48.920	2.411	46.769	2.678	5
56	56.143	1.701	53.572	1.925	51.158	2.163	48.883	2.416	46.734	2.683	4
57	56.099	1.705	53.531	1.929	51.119	2.167	48.846	2.420	46.699	2.688	3
58	56.054	1.709	53.489	1.933	51.080	2.171	48.809	2.424	46.664	2.692	2
59	56.010	1.712	53.448	1.937	51.041	2.175	48.773	2.429	46.630	2.697	1
60′	55.966	1.716	53.407	1.940	51.002	2.179	48.736	2.433	46.595	2.701	0′
	164°	**74°**	**163°**	**73°**	**162°**	**72°**	**161°**	**71°**	**160°**	**70°**	→
	344°	254°	343°	253°	342°	252°	341°	251°	340°	250°	→

S-Tables, P5

←	200°	290°	201°	291°	202°	292°	203°	293°	204°	294°	
←	20°	110°	21°	111°	22°	112°	23°	113°	24°	114°	
0′	46.595	2.701	44.567	2.985	42.642	3.283	40.812	3.597	39.069	3.927	60′
1	46.560	2.706	44.534	2.990	42.611	3.289	40.782	3.603	39.040	3.933	59
2	46.526	2.711	44.501	2.995	42.580	3.294	40.753	3.608	39.012	3.938	58
3	46.491	2.715	44.469	2.999	42.549	3.299	40.723	3.613	38.984	3.944	57
4	46.456	2.720	44.436	3.004	42.518	3.304	40.693	3.619	38.955	3.950	56
5	46.422	2.724	44.403	3.009	42.486	3.309	40.664	3.624	38.927	3.955	55
6	46.387	2.729	44.370	3.014	42.455	3.314	40.634	3.630	38.899	3.961	54
7	46.353	2.734	44.337	3.019	42.424	3.319	40.604	3.635	38.871	3.966	53
8	46.318	2.738	44.305	3.024	42.393	3.324	40.575	3.640	38.842	3.972	52
9	46.284	2.743	44.272	3.029	42.362	3.330	40.545	3.646	38.814	3.978	51
10	46.249	2.748	44.239	3.034	42.331	3.335	40.516	3.651	38.786	3.983	50
11	46.215	2.752	44.207	3.038	42.300	3.340	40.486	3.657	38.758	3.989	49
12	46.181	2.757	44.174	3.043	42.269	3.345	40.457	3.662	38.730	3.995	48
13	46.146	2.762	44.142	3.048	42.238	3.350	40.427	3.667	38.702	4.000	47
14	46.112	2.766	44.109	3.053	42.207	3.355	40.398	3.673	38.674	4.006	46
15	46.078	2.771	44.077	3.058	42.176	3.360	40.368	3.678	38.646	4.012	45
16	46.044	2.776	44.044	3.063	42.146	3.366	40.339	3.684	38.618	4.018	44
17	46.009	2.780	44.012	3.068	42.115	3.371	40.310	3.689	38.590	4.023	43
18	45.975	2.785	43.979	3.073	42.084	3.376	40.280	3.695	38.562	4.029	42
19	45.941	2.790	43.947	3.078	42.053	3.381	40.251	3.700	38.534	4.035	41
20	45.907	2.794	43.915	3.083	42.022	3.386	40.222	3.706	38.506	4.040	40
21	45.873	2.799	43.882	3.088	41.992	3.392	40.192	3.711	38.478	4.046	39
22	45.839	2.804	43.850	3.093	41.961	3.397	40.163	3.716	38.450	4.052	38
23	45.805	2.808	43.818	3.097	41.930	3.402	40.134	3.722	38.422	4.058	37
24	45.771	2.813	43.785	3.102	41.900	3.407	40.105	3.727	38.394	4.063	36
25	45.737	2.818	43.753	3.107	41.869	3.412	40.076	3.733	38.366	4.069	35
26	45.703	2.822	43.721	3.112	41.838	3.418	40.046	3.738	38.338	4.075	34
27	45.669	2.827	43.689	3.117	41.808	3.423	40.017	3.744	38.311	4.080	33
28	45.635	2.832	43.657	3.122	41.777	3.428	39.988	3.749	38.283	4.086	32
29	45.601	2.837	43.625	3.127	41.747	3.433	39.959	3.755	38.255	4.092	31
30	45.568	2.841	43.592	3.132	41.716	3.438	39.930	3.760	38.227	4.098	30
31	45.534	2.846	43.560	3.137	41.686	3.444	39.901	3.766	38.200	4.103	29
32	45.500	2.851	43.528	3.142	41.655	3.449	39.872	3.771	38.172	4.109	28
33	45.466	2.855	43.496	3.147	41.625	3.454	39.843	3.777	38.144	4.115	27
34	45.433	2.860	43.464	3.152	41.594	3.459	39.814	3.782	38.117	4.121	26
35	45.399	2.865	43.432	3.157	41.564	3.465	39.785	3.788	38.089	4.127	25
36	45.365	2.870	43.401	3.162	41.534	3.470	39.756	3.793	38.061	4.132	24
37	45.332	2.874	43.369	3.167	41.503	3.475	39.727	3.799	38.034	4.138	23
38	45.298	2.879	43.337	3.172	41.473	3.480	39.698	3.804	38.006	4.144	22
39	45.265	2.884	43.305	3.177	41.443	3.486	39.670	3.810	37.979	4.150	21
40	45.231	2.889	43.273	3.182	41.412	3.491	39.641	3.815	37.951	4.155	20
41	45.198	2.893	43.241	3.187	41.382	3.496	39.612	3.821	37.924	4.161	19
42	45.164	2.898	43.210	3.192	41.352	3.502	39.583	3.826	37.896	4.167	18
43	45.131	2.903	43.178	3.197	41.322	3.507	39.554	3.832	37.869	4.173	17
44	45.097	2.908	43.146	3.202	41.292	3.512	39.526	3.838	37.841	4.179	16
45	45.064	2.913	43.114	3.207	41.261	3.517	39.497	3.843	37.814	4.185	15
46	45.031	2.917	43.083	3.212	41.231	3.523	39.468	3.849	37.787	4.190	14
47	44.997	2.922	43.051	3.217	41.201	3.528	39.439	3.854	37.759	4.196	13
48	44.964	2.927	43.020	3.222	41.171	3.533	39.411	3.860	37.732	4.202	12
49	44.931	2.932	42.988	3.228	41.141	3.539	39.382	3.865	37.704	4.208	11
50	44.898	2.937	42.956	3.233	41.111	3.544	39.354	3.871	37.677	4.214	10
51	44.864	2.941	42.925	3.238	41.081	3.549	39.325	3.877	37.650	4.220	9
52	44.831	2.946	42.893	3.243	41.051	3.555	39.296	3.882	37.623	4.225	8
53	44.798	2.951	42.862	3.248	41.021	3.560	39.268	3.888	37.595	4.231	7
54	44.765	2.956	42.831	3.253	40.991	3.565	39.239	3.893	37.568	4.237	6
55	44.732	2.961	42.799	3.258	40.961	3.571	39.211	3.899	37.541	4.243	5
56	44.699	2.965	42.768	3.263	40.931	3.576	39.182	3.905	37.514	4.249	4
57	44.666	2.970	42.736	3.268	40.902	3.581	39.154	3.910	37.487	4.255	3
58	44.633	2.975	42.705	3.273	40.872	3.587	39.125	3.916	37.459	4.261	2
59	44.600	2.980	42.674	3.278	40.842	3.592	39.097	3.921	37.432	4.267	1
60′	44.567	2.985	42.642	3.283	40.812	3.597	39.069	3.927	37.405	4.272	0′
	159°	69°	158°	68°	157°	67°	156°	66°	155°	65°	→
	339°	249°	338°	248°	337°	247°	336°	246°	335°	245°	→

| ← | 205° | 295° | 206° | 296° | 207° | 297° | 208° | 298° | 209° | 299° | |
←	25°	115°	26°	116°	27°	117°	28°	118°	29°	119°	
0′	37.405	4.272	35.816	4.634	34.295	5.012	32.839	5.406	31.443	5.818	60′
1	37.378	4.278	35.790	4.640	34.271	5.018	32.815	5.413	31.420	5.825	59
2	37.351	4.284	35.764	4.646	34.246	5.025	32.792	5.420	31.397	5.832	58
3	37.324	4.290	35.738	4.652	34.221	5.031	32.768	5.427	31.375	5.839	57
4	37.297	4.296	35.712	4.659	34.196	5.038	32.744	5.433	31.352	5.846	56
5	37.270	4.302	35.687	4.665	34.172	5.044	32.721	5.440	31.329	5.853	55
6	37.243	4.308	35.661	4.671	34.147	5.051	32.697	5.447	31.306	5.860	54
7	37.216	4.314	35.635	4.677	34.122	5.057	32.673	5.454	31.284	5.867	53
8	37.189	4.320	35.609	4.683	34.098	5.064	32.650	5.460	31.261	5.874	52
9	37.162	4.326	35.583	4.690	34.073	5.070	32.626	5.467	31.238	5.881	51
10	37.135	4.332	35.558	4.696	34.048	5.077	32.602	5.474	31.216	5.888	50
11	37.108	4.337	35.532	4.702	34.024	5.083	32.579	5.481	31.193	5.895	49
12	37.082	4.343	35.506	4.708	33.999	5.089	32.555	5.487	31.171	5.902	48
13	37.055	4.349	35.481	4.714	33.975	5.096	32.532	5.494	31.148	5.910	47
14	37.028	4.355	35.455	4.721	33.950	5.102	32.508	5.501	31.125	5.917	46
15	37.001	4.361	35.429	4.727	33.925	5.109	32.485	5.508	31.103	5.924	45
16	36.974	4.367	35.404	4.733	33.901	5.115	32.461	5.515	31.080	5.931	44
17	36.948	4.373	35.378	4.739	33.876	5.122	32.438	5.521	31.058	5.938	43
18	36.921	4.379	35.353	4.746	33.852	5.129	32.414	5.528	31.035	5.945	42
19	36.894	4.385	35.327	4.752	33.827	5.135	32.391	5.535	31.013	5.952	41
20	36.867	4.391	35.302	4.758	33.803	5.142	32.367	5.542	30.990	5.959	40
21	36.841	4.397	35.276	4.764	33.779	5.148	32.344	5.549	30.968	5.966	39
22	36.814	4.403	35.251	4.771	33.754	5.155	32.320	5.555	30.945	5.973	38
23	36.787	4.409	35.225	4.777	33.730	5.161	32.297	5.562	30.923	5.980	37
24	36.761	4.415	35.200	4.783	33.705	5.168	32.274	5.569	30.900	5.988	36
25	36.734	4.421	35.174	4.789	33.681	5.174	32.250	5.576	30.878	5.995	35
26	36.708	4.427	35.149	4.796	33.657	5.181	32.227	5.583	30.856	6.002	34
27	36.681	4.433	35.123	4.802	33.632	5.187	32.204	5.590	30.833	6.009	33
28	36.655	4.439	35.098	4.808	33.608	5.194	32.180	5.596	30.811	6.016	32
29	36.628	4.445	35.073	4.815	33.584	5.201	32.157	5.603	30.788	6.023	31
30	36.602	4.451	35.047	4.821	33.559	5.207	32.134	5.610	30.766	6.030	30
31	36.575	4.457	35.022	4.827	33.535	5.214	32.110	5.617	30.744	6.037	29
32	36.549	4.463	34.997	4.833	33.511	5.220	32.087	5.624	30.722	6.045	28
33	36.522	4.469	34.971	4.840	33.487	5.227	32.064	5.631	30.699	6.052	27
34	36.496	4.475	34.946	4.846	33.463	5.233	32.041	5.638	30.677	6.059	26
35	36.469	4.481	34.921	4.852	33.438	5.240	32.018	5.644	30.655	6.066	25
36	36.443	4.487	34.896	4.859	33.414	5.247	31.994	5.651	30.632	6.073	24
37	36.417	4.493	34.870	4.865	33.390	5.253	31.971	5.658	30.610	6.080	23
38	36.390	4.500	34.845	4.871	33.366	5.260	31.948	5.665	30.588	6.088	22
39	36.364	4.506	34.820	4.878	33.342	5.266	31.925	5.672	30.566	6.095	21
40	36.338	4.512	34.795	4.884	33.318	5.273	31.902	5.679	30.544	6.102	20
41	36.311	4.518	34.770	4.890	33.294	5.280	31.879	5.686	30.521	6.109	19
42	36.285	4.524	34.745	4.897	33.269	5.286	31.856	5.693	30.499	6.116	18
43	36.259	4.530	34.719	4.903	33.245	5.293	31.833	5.700	30.477	6.124	17
44	36.233	4.536	34.694	4.910	33.221	5.300	31.810	5.707	30.455	6.131	16
45	36.207	4.542	34.669	4.916	33.197	5.306	31.787	5.714	30.433	6.138	15
46	36.180	4.548	34.644	4.922	33.173	5.313	31.764	5.720	30.411	6.145	14
47	36.154	4.554	34.619	4.929	33.149	5.320	31.741	5.727	30.389	6.153	13
48	36.128	4.560	34.594	4.935	33.125	5.326	31.718	5.734	30.367	6.160	12
49	36.102	4.566	34.569	4.941	33.101	5.333	31.695	5.741	30.345	6.167	11
50	36.076	4.573	34.544	4.948	33.078	5.340	31.672	5.748	30.323	6.174	10
51	36.050	4.579	34.519	4.954	33.054	5.346	31.649	5.755	30.301	6.181	9
52	36.024	4.585	34.494	4.961	33.030	5.353	31.626	5.762	30.279	6.189	8
53	35.998	4.591	34.469	4.967	33.006	5.360	31.603	5.769	30.257	6.196	7
54	35.972	4.597	34.444	4.973	32.982	5.366	31.580	5.776	30.235	6.203	6
55	35.946	4.603	34.420	4.980	32.958	5.373	31.557	5.783	30.213	6.211	5
56	35.920	4.609	34.395	4.986	32.934	5.380	31.534	5.790	30.191	6.218	4
57	35.894	4.616	34.370	4.993	32.910	5.386	31.511	5.797	30.169	6.225	3
58	35.868	4.622	34.345	4.999	32.887	5.393	31.489	5.804	30.147	6.232	2
59	35.842	4.628	34.320	5.005	32.863	5.400	31.466	5.811	30.125	6.240	1
60′	35.816	4.634	34.295	5.012	32.839	5.406	31.443	5.818	30.103	6.247	0′
	154°	64°	153°	63°	152°	62°	151°	61°	150°	60°	→
	334°	244°	333°	243°	332°	242°	331°	241°	330°	240°	→

S-Tables, P7

	210°	300°	211°	301°	212°	302°	213°	303°	214°	304°	
	30°	120°	31°	121°	32°	122°	33°	123°	34°	124°	
0′	30.103	6.247	28.816	6.693	27.579	7.158	26.389	7.641	25.244	8.143	60′
1	30.081	6.254	28.795	6.701	27.559	7.166	26.370	7.649	25.225	8.151	59
2	30.059	6.262	28.774	6.709	27.539	7.174	26.350	7.657	25.206	8.160	58
3	30.037	6.269	28.753	6.716	27.518	7.182	26.331	7.665	25.188	8.168	57
4	30.016	6.276	28.732	6.724	27.498	7.190	26.311	7.674	25.169	8.177	56
5	29.994	6.283	28.711	6.731	27.478	7.197	26.292	7.682	25.150	8.185	55
6	29.972	6.291	28.690	6.739	27.458	7.205	26.273	7.690	25.132	8.194	54
7	29.950	6.298	28.669	6.747	27.438	7.213	26.253	7.698	25.113	8.202	53
8	29.928	6.305	28.648	6.754	27.418	7.221	26.234	7.707	25.094	8.211	52
9	29.907	6.313	28.627	6.762	27.398	7.229	26.215	7.715	25.076	8.219	51
10	29.885	6.320	28.607	6.770	27.378	7.237	26.195	7.723	25.057	8.228	50
11	29.863	6.327	28.586	6.777	27.357	7.245	26.176	7.731	25.039	8.237	49
12	29.842	6.335	28.565	6.785	27.337	7.253	26.157	7.740	25.020	8.245	48
13	29.820	6.342	28.544	6.793	27.317	7.261	26.137	7.748	25.001	8.254	47
14	29.798	6.350	28.523	6.800	27.297	7.269	26.118	7.756	24.983	8.262	46
15	29.776	6.357	28.502	6.808	27.277	7.277	26.099	7.764	24.964	8.271	45
16	29.755	6.364	28.481	6.816	27.257	7.285	26.079	7.773	24.946	8.280	44
17	29.733	6.372	28.461	6.823	27.237	7.293	26.060	7.781	24.927	8.288	43
18	29.712	6.379	28.440	6.831	27.217	7.301	26.041	7.789	24.909	8.297	42
19	29.690	6.386	28.419	6.839	27.197	7.309	26.022	7.798	24.890	8.305	41
20	29.668	6.394	28.398	6.846	27.177	7.317	26.003	7.806	24.872	8.314	40
21	29.647	6.401	28.378	6.854	27.157	7.325	25.983	7.814	24.853	8.323	39
22	29.625	6.409	28.357	6.862	27.137	7.333	25.964	7.823	24.835	8.331	38
23	29.604	6.416	28.336	6.869	27.118	7.341	25.945	7.831	24.816	8.340	37
24	29.582	6.423	28.315	6.877	27.098	7.349	25.926	7.839	24.798	8.349	36
25	29.561	6.431	28.295	6.885	27.078	7.357	25.907	7.848	24.779	8.357	35
26	29.539	6.438	28.274	6.892	27.058	7.365	25.888	7.856	24.761	8.366	34
27	29.518	6.446	28.253	6.900	27.038	7.373	25.868	7.864	24.742	8.375	33
28	29.496	6.453	28.233	6.908	27.018	7.381	25.849	7.873	24.724	8.383	32
29	29.475	6.461	28.212	6.916	26.998	7.389	25.830	7.881	24.706	8.392	31
30	29.453	6.468	28.192	6.923	26.978	7.397	25.811	7.889	24.687	8.401	30
31	29.432	6.475	28.171	6.931	26.959	7.405	25.792	7.898	24.669	8.409	29
32	29.410	6.483	28.150	6.939	26.939	7.413	25.773	7.906	24.650	8.418	28
33	29.389	6.490	28.130	6.947	26.919	7.421	25.754	7.914	24.632	8.427	27
34	29.367	6.498	28.109	6.954	26.899	7.429	25.735	7.923	24.614	8.435	26
35	29.346	6.505	28.089	6.962	26.879	7.437	25.716	7.931	24.595	8.444	25
36	29.325	6.513	28.068	6.970	26.860	7.445	25.697	7.940	24.577	8.453	24
37	29.303	6.520	28.048	6.978	26.840	7.454	25.678	7.948	24.559	8.462	23
38	29.282	6.528	28.027	6.986	26.820	7.462	25.659	7.956	24.541	8.470	22
39	29.261	6.535	28.007	6.993	26.800	7.470	25.640	7.965	24.522	8.479	21
40	29.239	6.543	27.986	7.001	26.781	7.478	25.621	7.973	24.504	8.488	20
41	29.218	6.550	27.966	7.009	26.761	7.486	25.602	7.982	24.486	8.496	19
42	29.197	6.558	27.945	7.017	26.741	7.494	25.583	7.990	24.467	8.505	18
43	29.176	6.565	27.925	7.024	26.722	7.502	25.564	7.998	24.449	8.514	17
44	29.154	6.573	27.904	7.032	26.702	7.510	25.545	8.007	24.431	8.523	16
45	29.133	6.580	27.884	7.040	26.682	7.518	25.526	8.015	24.413	8.531	15
46	29.112	6.588	27.863	7.048	26.663	7.527	25.507	8.024	24.395	8.540	14
47	29.091	6.595	27.843	7.056	26.643	7.535	25.488	8.032	24.376	8.549	13
48	29.069	6.603	27.823	7.064	26.623	7.543	25.469	8.041	24.358	8.558	12
49	29.048	6.610	27.802	7.071	26.604	7.551	25.451	8.049	24.340	8.567	11
50	29.027	6.618	27.782	7.079	26.584	7.559	25.432	8.058	24.322	8.575	10
51	29.006	6.625	27.762	7.087	26.565	7.567	25.413	8.066	24.304	8.584	9
52	28.985	6.633	27.741	7.095	26.545	7.575	25.394	8.075	24.286	8.593	8
53	28.964	6.640	27.721	7.103	26.526	7.584	25.375	8.083	24.267	8.602	7
54	28.942	6.648	27.701	7.111	26.506	7.592	25.356	8.092	24.249	8.611	6
55	28.921	6.656	27.680	7.119	26.487	7.600	25.338	8.100	24.231	8.619	5
56	28.900	6.663	27.660	7.126	26.467	7.608	25.319	8.109	24.213	8.628	4
57	28.879	6.671	27.640	7.134	26.448	7.616	25.300	8.117	24.195	8.637	3
58	28.858	6.678	27.620	7.142	26.428	7.624	25.281	8.126	24.177	8.646	2
59	28.837	6.686	27.599	7.150	26.409	7.633	25.263	8.134	24.159	8.655	1
60′	28.816	6.693	27.579	7.158	26.389	7.641	25.244	8.143	24.141	8.664	0′
	149°	59°	148°	58°	147°	57°	146°	56°	145°	55°	→
	329°	239°	328°	238°	327°	237°	326°	236°	325°	235°	→

S-Tables, P8

←	215°	305°	216°	306°	217°	307°	218°	308°	219°	309°	
←	35°	125°	36°	126°	37°	127°	38°	128°	39°	129°	
0′	24.141	8.664	23.078	9.204	22.054	9.765	21.066	10.347	20.113	10.950	60′
1	24.123	8.672	23.061	9.213	22.037	9.775	21.050	10.357	20.097	10.960	59
2	24.105	8.681	23.043	9.223	22.020	9.784	21.034	10.367	20.082	10.970	58
3	24.087	8.690	23.026	9.232	22.003	9.794	21.017	10.376	20.066	10.980	57
4	24.069	8.699	23.009	9.241	21.987	9.803	21.001	10.386	20.051	10.991	56
5	24.051	8.708	22.991	9.250	21.970	9.813	20.985	10.396	20.035	11.001	55
6	24.033	8.717	22.974	9.259	21.953	9.822	20.969	10.406	20.019	11.011	54
7	24.015	8.726	22.957	9.269	21.937	9.832	20.953	10.416	20.004	11.021	53
8	23.997	8.734	22.939	9.278	21.920	9.841	20.937	10.426	19.988	11.032	52
9	23.979	8.743	22.922	9.287	21.903	9.851	20.921	10.436	19.973	11.042	51
10	23.961	8.752	22.905	9.296	21.887	9.861	20.905	10.446	19.957	11.052	50
11	23.943	8.761	22.888	9.306	21.870	9.870	20.889	10.456	19.942	11.063	49
12	23.925	8.770	22.870	9.315	21.853	9.880	20.872	10.466	19.926	11.073	48
13	23.907	8.779	22.853	9.324	21.837	9.889	20.856	10.476	19.911	11.083	47
14	23.889	8.788	22.836	9.333	21.820	9.899	20.840	10.486	19.895	11.094	46
15	23.872	8.797	22.819	9.343	21.803	9.909	20.824	10.495	19.880	11.104	45
16	23.854	8.806	22.801	9.352	21.787	9.918	20.808	10.505	19.864	11.114	44
17	23.836	8.815	22.784	9.361	21.770	9.928	20.792	10.515	19.849	11.125	43
18	23.818	8.824	22.767	9.370	21.754	9.937	20.776	10.525	19.834	11.135	42
19	23.800	8.833	22.750	9.380	21.737	9.947	20.760	10.535	19.818	11.145	41
20	23.782	8.842	22.733	9.389	21.720	9.957	20.744	10.545	19.803	11.156	40
21	23.764	8.851	22.715	9.398	21.704	9.966	20.728	10.555	19.787	11.166	39
22	23.747	8.859	22.698	9.408	21.687	9.976	20.712	10.565	19.772	11.176	38
23	23.729	8.868	22.681	9.417	21.671	9.986	20.696	10.575	19.756	11.187	37
24	23.711	8.877	22.664	9.426	21.654	9.995	20.681	10.585	19.741	11.197	36
25	23.693	8.886	22.647	9.435	21.638	10.005	20.665	10.595	19.726	11.207	35
26	23.676	8.895	22.630	9.445	21.621	10.015	20.649	10.605	19.710	11.218	34
27	23.658	8.904	22.613	9.454	21.605	10.024	20.633	10.615	19.695	11.228	33
28	23.640	8.913	22.595	9.463	21.588	10.034	20.617	10.625	19.680	11.239	32
29	23.622	8.922	22.578	9.473	21.572	10.044	20.601	10.635	19.664	11.249	31
30	23.605	8.931	22.561	9.482	21.555	10.053	20.585	10.646	19.649	11.259	30
31	23.587	8.940	22.544	9.491	21.539	10.063	20.569	10.656	19.634	11.270	29
32	23.569	8.949	22.527	9.501	21.522	10.073	20.553	10.666	19.618	11.280	28
33	23.552	8.958	22.510	9.510	21.506	10.082	20.537	10.676	19.603	11.291	27
34	23.534	8.967	22.493	9.520	21.490	10.092	20.522	10.686	19.588	11.301	26
35	23.516	8.977	22.476	9.529	21.473	10.102	20.506	10.696	19.572	11.312	25
36	23.499	8.986	22.459	9.538	21.457	10.112	20.490	10.706	19.557	11.322	24
37	23.481	8.995	22.442	9.548	21.440	10.121	20.474	10.716	19.542	11.332	23
38	23.463	9.004	22.425	9.557	21.424	10.131	20.458	10.726	19.527	11.343	22
39	23.446	9.013	22.408	9.566	21.408	10.141	20.443	10.736	19.511	11.353	21
40	23.428	9.022	22.391	9.576	21.391	10.151	20.427	10.746	19.496	11.364	20
41	23.410	9.031	22.374	9.585	21.375	10.160	20.411	10.756	19.481	11.374	19
42	23.393	9.040	22.357	9.595	21.358	10.170	20.395	10.767	19.466	11.385	18
43	23.375	9.049	22.340	9.604	21.342	10.180	20.379	10.777	19.451	11.395	17
44	23.358	9.058	22.323	9.614	21.326	10.190	20.364	10.787	19.435	11.406	16
45	23.340	9.067	22.306	9.623	21.309	10.199	20.348	10.797	19.420	11.416	15
46	23.323	9.076	22.289	9.632	21.293	10.209	20.332	10.807	19.405	11.427	14
47	23.305	9.085	22.273	9.642	21.277	10.219	20.316	10.817	19.390	11.437	13
48	23.288	9.094	22.256	9.651	21.261	10.229	20.301	10.827	19.375	11.448	12
49	23.270	9.104	22.239	9.661	21.244	10.239	20.285	10.838	19.359	11.458	11
50	23.253	9.113	22.222	9.670	21.228	10.248	20.269	10.848	19.344	11.469	10
51	23.235	9.122	22.205	9.680	21.212	10.258	20.254	10.858	19.329	11.479	9
52	23.218	9.131	22.188	9.689	21.196	10.268	20.238	10.868	19.314	11.490	8
53	23.200	9.140	22.171	9.699	21.179	10.278	20.222	10.878	19.299	11.501	7
54	23.183	9.149	22.155	9.708	21.163	10.288	20.207	10.888	19.284	11.511	6
55	23.165	9.158	22.138	9.718	21.147	10.297	20.191	10.899	19.269	11.522	5
56	23.148	9.168	22.121	9.727	21.131	10.307	20.175	10.909	19.254	11.532	4
57	23.130	9.177	22.104	9.737	21.114	10.317	20.160	10.919	19.238	11.543	3
58	23.113	9.186	22.087	9.746	21.098	10.327	20.144	10.929	19.223	11.553	2
59	23.096	9.195	22.070	9.756	21.082	10.337	20.128	10.939	19.208	11.564	1
60′	23.078	9.204	22.054	9.765	21.066	10.347	20.113	10.950	19.193	11.575	0′
	144°	54°	143°	53°	142°	52°	141°	51°	140°	50°	→
	324°	234°	323°	233°	322°	232°	321°	231°	320°	230°	→

S-Tables, P9

←	220°	310°	221°	311°	222°	312°	223°	313°	224°	314°	
←	40°	130°	41°	131°	42°	132°	43°	133°	44°	134°	
0'	19.193	11.575	18.306	12.222	17.449	12.893	16.622	13.587	15.823	14.307	60'
1	19.178	11.585	18.291	12.233	17.435	12.904	16.608	13.599	15.810	14.319	59
2	19.163	11.596	18.277	12.244	17.421	12.915	16.595	13.611	15.797	14.331	58
3	19.148	11.606	18.262	12.255	17.407	12.927	16.581	13.623	15.784	14.343	57
4	19.133	11.617	18.248	12.266	17.393	12.938	16.568	13.634	15.771	14.355	56
5	19.118	11.628	18.233	12.277	17.379	12.950	16.554	13.646	15.758	14.368	55
6	19.103	11.638	18.219	12.288	17.365	12.961	16.541	13.658	15.745	14.380	54
7	19.088	11.649	18.204	12.299	17.351	12.972	16.527	13.670	15.732	14.392	53
8	19.073	11.660	18.190	12.310	17.337	12.984	16.514	13.682	15.718	14.404	52
9	19.058	11.670	18.175	12.321	17.323	12.995	16.500	13.694	15.705	14.417	51
10	19.043	11.681	18.161	12.332	17.309	13.007	16.487	13.705	15.692	14.429	50
11	19.028	11.692	18.146	12.343	17.295	13.018	16.473	13.717	15.679	14.441	49
12	19.013	11.702	18.132	12.354	17.281	13.030	16.460	13.729	15.666	14.453	48
13	18.998	11.713	18.118	12.365	17.267	13.041	16.446	13.741	15.653	14.466	47
14	18.983	11.724	18.103	12.376	17.253	13.053	16.433	13.753	15.640	14.478	46
15	18.968	11.734	18.089	12.387	17.239	13.064	16.419	13.765	15.628	14.490	45
16	18.954	11.745	18.074	12.399	17.225	13.075	16.406	13.777	15.615	14.503	44
17	18.939	11.756	18.060	12.410	17.212	13.087	16.393	13.788	15.602	14.515	43
18	18.924	11.766	18.046	12.421	17.198	13.098	16.379	13.800	15.589	14.527	42
19	18.909	11.777	18.031	12.432	17.184	13.110	16.366	13.812	15.576	14.540	41
20	18.894	11.788	18.017	12.443	17.170	13.121	16.352	13.824	15.563	14.552	40
21	18.879	11.799	18.002	12.454	17.156	13.133	16.339	13.836	15.550	14.564	39
22	18.864	11.809	17.988	12.465	17.142	13.144	16.326	13.848	15.537	14.577	38
23	18.849	11.820	17.974	12.476	17.128	13.156	16.312	13.860	15.524	14.589	37
24	18.834	11.831	17.959	12.487	17.115	13.168	16.299	13.872	15.511	14.601	36
25	18.820	11.842	17.945	12.499	17.101	13.179	16.285	13.884	15.498	14.614	35
26	18.805	11.852	17.931	12.510	17.087	13.191	16.272	13.896	15.485	14.626	34
27	18.790	11.863	17.916	12.521	17.073	13.202	16.259	13.908	15.472	14.639	33
28	18.775	11.874	17.902	12.532	17.059	13.214	16.245	13.920	15.460	14.651	32
29	18.760	11.885	17.888	12.543	17.045	13.225	16.232	13.932	15.447	14.663	31
30	18.746	11.895	17.874	12.554	17.032	13.237	16.219	13.944	15.434	14.676	30
31	18.731	11.906	17.859	12.566	17.018	13.248	16.206	13.956	15.421	14.688	29
32	18.716	11.917	17.845	12.577	17.004	13.260	16.192	13.968	15.408	14.701	28
33	18.701	11.928	17.831	12.588	16.990	13.272	16.179	13.980	15.395	14.713	27
34	18.686	11.939	17.817	12.599	16.977	13.283	16.166	13.992	15.382	14.725	26
35	18.672	11.949	17.802	12.610	16.963	13.295	16.152	14.004	15.370	14.738	25
36	18.657	11.960	17.788	12.622	16.949	13.306	16.139	14.016	15.357	14.750	24
37	18.642	11.971	17.774	12.633	16.935	13.318	16.126	14.028	15.344	14.763	23
38	18.628	11.982	17.760	12.644	16.922	13.330	16.113	14.040	15.331	14.775	22
39	18.613	11.993	17.745	12.655	16.908	13.341	16.099	14.052	15.318	14.788	21
40	18.598	12.004	17.731	12.666	16.894	13.353	16.086	14.064	15.306	14.800	20
41	18.583	12.014	17.717	12.678	16.881	13.365	16.073	14.076	15.293	14.813	19
42	18.569	12.025	17.703	12.689	16.867	13.376	16.060	14.088	15.280	14.825	18
43	18.554	12.036	17.689	12.700	16.853	13.388	16.046	14.100	15.267	14.838	17
44	18.539	12.047	17.674	12.711	16.839	13.400	16.033	14.112	15.255	14.850	16
45	18.525	12.058	17.660	12.723	16.826	13.411	16.020	14.124	15.242	14.863	15
46	18.510	12.069	17.646	12.734	16.812	13.423	16.007	14.136	15.229	14.875	14
47	18.495	12.080	17.632	12.745	16.798	13.435	15.994	14.149	15.216	14.888	13
48	18.481	12.091	17.618	12.757	16.785	13.446	15.980	14.161	15.204	14.900	12
49	18.466	12.102	17.604	12.768	16.771	13.458	15.967	14.173	15.191	14.913	11
50	18.451	12.112	17.590	12.779	16.758	13.470	15.954	14.185	15.178	14.926	10
51	18.437	12.123	17.576	12.791	16.744	13.481	15.941	14.197	15.166	14.938	9
52	18.422	12.134	17.561	12.802	16.730	13.493	15.928	14.209	15.153	14.951	8
53	18.408	12.145	17.547	12.813	16.717	13.505	15.915	14.221	15.140	14.963	7
54	18.393	12.156	17.533	12.825	16.703	13.517	15.902	14.233	15.127	14.976	6
55	18.379	12.167	17.519	12.836	16.690	13.528	15.888	14.246	15.115	14.988	5
56	18.364	12.178	17.505	12.847	16.676	13.540	15.875	14.258	15.102	15.001	4
57	18.349	12.189	17.491	12.859	16.662	13.552	15.862	14.270	15.089	15.014	3
58	18.335	12.200	17.477	12.870	16.649	13.564	15.849	14.282	15.077	15.026	2
59	18.320	12.211	17.463	12.881	16.635	13.575	15.836	14.294	15.064	15.039	1
60'	18.306	12.222	17.449	12.893	16.622	13.587	15.823	14.307	15.052	15.051	0'
	139°	49°	138°	48°	137°	47°	136°	46°	135°	45°	→
	319°	229°	318°	228°	317°	227°	316°	226°	315°	225°	→

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