

# Navigation Course

This is an advanced [online course on marine navigation](#), providing you with the “conditio sine qua non” of offshore sailing. Nowadays most sailors tend to rely on modern equipment like differential GPS or Radar to navigate them through hazardous waters. Not only is such reliance unwanted and possibly dangerous, also the act of navigating by yourself is actually a lot of fun.

## What is navigation?

“[Navigare necesse est, vivere non est necesse](#)” is latin for: to sail is vital, to live is not. This phrase tells us that both sailing and the “conditio” of positioning are highly intertwined. Indeed, the art of navigation enables you to set a course and sail to your destination by using only nautical charts, a compass and your common sense.

The aim of this course is to teach you how to navigate safely while using the minimum of resources: methods that have been in use since the Middle Ages, and are still applied by the professionals [L.I.](#) This course greatly extends on - for instance - the ASA courses and gives you the insight and feel of a seasoned navigator.

Enjoy!

This is chapter 0: Use the moving anchor logo on the right of this page to navigate through the course. Alternatively, click on the links below to study the chapters.

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**Chapter 3 - Compass**

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# Positions

## Longitude and latitude

The earth can be regarded as a spherical object, and since we're dealing with a 3-dimensional shape we need coordinates of a different form than the usual x- and y-axes. Though adding an extra z-axis would make sense for submarines, we will most likely be found on the surface of this sphere while using another system of coordinates, that covers our planet with imaginary lines called **meridians** and **parallels**, see figure 1. All these lines together provide the grid which enables us to describe any position in **longitudes** and **latitudes**.

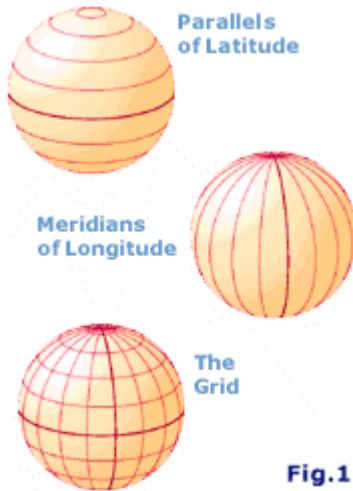


Fig.1

The obvious place to divide the Northern and Southern Hemispheres [\[1\]](#) was the equator. But the division of the Eastern and Western hemispheres was the source of much political turmoil. Greenwich (Great Britain) won, placing for example The Netherlands in the Eastern and Ireland in the Western Hemisphere.

It takes the earth 24 hours for a full rotation of 360°. Thus, every hour we rotate 15° longitude, see figure 2. When it is 12:00 Greenwich Mean Time (GMT) - anywhere in the world - it is 12:00 Local Time in Greenwich and 24:00 Local Time at the other side of the planet: 180° E or 180° W: the **date line**. Crossing this special meridian changes not only the hour but also the date.

The North Pole has a latitude of 90° N and the South Pole 90° S. The meridians cover twice this angle up to 180° W or E.

Meridians converge at the poles, whereas parallels run parallel to each other and never meet. All meridians and the equator - the biggest parallel - form **great circles**, and the remaining parallels form so-called **small circles**. A great circle divides the earth in two exact halves.

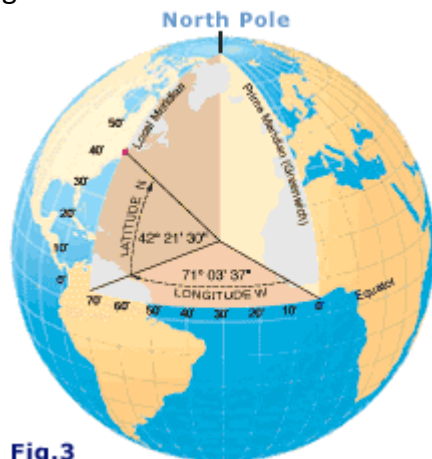


Fig.3

In figure 3 the position of Boston in the United States is shown using latitude and longitude in degrees, minutes and seconds:

42° 21' 30" N , 71° 03' 37" W

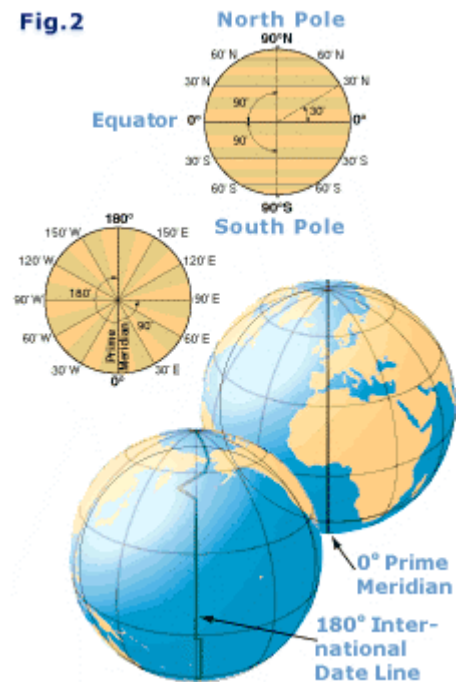
Most sailors will actually notate seconds in metric fractions of minutes:

42° 21',5 N , 71° 03',6 W or

42° 21'.5 N , 71° 03'.6 W [\[1\]](#)

On small scaled charts we want to be accurate within one minute or one nautical mile. On larger scaled charts the accuracy is more likely to be within a tenth of a mile (a

Fig.2



cable).

If the earth were a perfect sphere with a circumference of roughly 40000 kilometres all great circles - meridians plus the equator - would have the same length and could be used as a distance unit when divided into 360 degrees, or  $360^\circ \times 60' = 21600'$  minutes. In 1929, the international community agreed on the definition of 1 international nautical mile as 1852 metres, which is roughly the *average* length of one minute of latitude i.e. one minute of arc along a line of longitude (a meridian).


Or to put it shortly: **1 nm = 1'**

We are now able to describe any position in latitudes and longitudes. Moreover, we can state the distance between two of those positions using nautical miles or minutes. All we need now is a proper way to define speed. For that, sailors use **knots**, the number of nautical miles an hour.

## RYA & ASA sailing schools



To put this navigation course into practice a **Royal Yachting Association** or **American Sailing Association** approved sailing course is recommended.

## A little History

Mariners during the 15th century relied on charts called "portolans" to assist them on their voyages. *Portolan* comes from the Italian word *portolani* , which were medieval pilot books.

The portolans contained maps of coastlines, locations of harbours, river mouths, and man-made features visible from the sea. They were a compilation of centuries of seafarer observations. As sailors' skills improved and the use of the **compass** was more widespread, portolans improved in accuracy.



Also Columbus  used these portolans on his journeys. Portuguese chart makers added the meridian line, a point useful for latitude sailing as well as for navigating solely by compass. A geographic feature could now be located through the use of its distance in degrees of latitude from a ship's point of departure. Note that the use of latitude and longitude was understood since the time of Ptolemy , the second century CE.

During the fifteenth century Portugal led the European world in sea exploration. The golden age of discovery for Portugal lasted almost a century until the Dutch eventually seized their trade routes from them.

As we move to the next chapter of this course we enter the sixteenth century when the Mercator chart was invented.

## Overview

**Parallels:** Circles parallel to the equator, ranging from  $0^\circ$  to  $90^\circ$  N or S. Only the equator is a great circle.

**Meridians:** half-circles converging at the poles, ranging from  $0^\circ$  to  $180^\circ$  E or W. Each pair of opposing meridians forms a great circle.

**Prime meridian:**  $0^\circ$  or the Greenwich meridian which - together with the date line meridian - divides the Western and Eastern hemispheres.

**Great circle:** The intersection of a sphere and a plane that passes through the sphere's centre.

**Small circle:** The intersection of a sphere and a plane that doesn't pass through the sphere's centre.

**Time zones:** By convention 24 zones, each 15° longitude wide. Hence, noon at Greenwich gives midnight at 180° E.

**GMT:** Greenwich Mean Time, UTC or Zulu, which is the local time at Greenwich. Antonym: Local time elsewhere. For example, local time in Athens = GMT + 2.

**Date line:** The 180° meridian which extends from or is opposite to the prime meridian. Here, not only the hour changes when crossing the meridian, but also the date.

**Latitude:** Position property defined by the number of degrees north or south of the equator, varies from 0° to 90°.

**Longitude:** Position property defined by the number of degrees east or west of the prime meridian, varies from 0° to 180°.

**Position:** Latitude first and longitude second. For example: Athens in Greece 37° 58' N , 23° 43' E.

**Nautical mile:** One nm is one minute (') on the vertical scale on the chart. 1' equals 1852 metres. Nautical miles are divided into 10 cables.

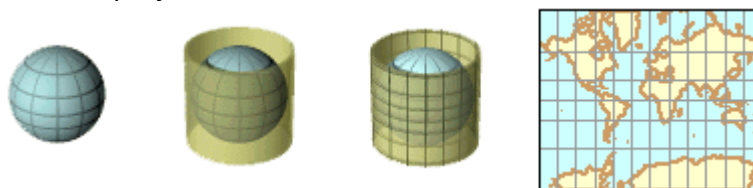
**Knots:** Nautical miles per hour.

## Projections

The nautical chart is a 2-dimensional representation of a 3-dimensional world. And although this results in various distortions, as long as two requirements are met we can use this image for navigational purposes.

1. The angles between three objects in the chart should be the same as the angles between the real objects which they represent.
2. A straight course should appear as a straight line in the chart.

To fulfil these demands a nautical chart requires parallels and meridians that are both straight and parallel. Moreover, the meridians will need to be perpendicular to the parallels. A well known method to create such a chart is called the **Mercator projection** after Gerard “**Mercator**” Kremer [\[1\]](#), a Flemish scholar who studied in 's Hertogenbosch and Leuven and who invented his famous projection in 1569.



The Mercator chart was designed for sailors and can be constructed by wrapping a cylinder around the planet so that it touches the equator. On this cylinder the surface of the earth is projected and finally the cylinder is cut open to yield our chart. But where the meridians converge on the globe they run parallel in the projection (see chart below), indicating the distortion. Look, for example, at a high parallel. The length of such a parallel on the globe is much smaller than the equator. Yet, on the chart they have exactly the same length creating a distortion which gets bigger nearer to the poles. The figure below shows us the construction of the Mercator projection. From this it is clear that only the vertical scales should be used for measuring distances.



The **vertical scale** depicted on the right demonstrates the distortion. The two little grey markers have the same size, the upper one measures only 0.71 degrees while the other measures 1.00 degrees. So, distances (in miles or in minutes) should not only be read on the vertical scale, but also at approximately the same height.

The **horizontal scale** is only valid for one latitude in the chart and can therefore only be used for the coordinates (a point, but not a line). If you divide the surface of the earth in eight pieces, and lift one out and project it, you end up



with the figure below. The result is that both A-A' and B-B' are now as long as the bottom of the chart and are "too long".

But there are of course other projections in use by sailors. An important one is the **Stereographic projection**, which is constructed by projecting on a flat plane instead of a cylinder. On this chart parallels appear as slightly curved and also the meridians converge at high latitudes. So, strictly speaking, a straight course will not appear as a straight line in the chart, but the parallels remain perpendicular to the meridians. Most often, distortions are scarcely noticed when this projection is used to chart a small area. Like the Mercator projection, the vertical scale represents a meridian and should be used for measuring distances.

Another projection is the **Gnomic projection** on which the meridians are again converging. But most importantly, the parallels are arcs of a circle while great circles appear as straight lines. On a sphere the shortest route between A and B is not a straight line but an arc (part of a great circle). Though this is also true when you – for example – cross a little bay, we use for simplification a **loxodrome** (a handy straight line on your Mercator chart which does not reflect your shortest route). On a Gnomic chart this same loxodrome is an arc, while your shortest route (a great circle) ends up as a straight line. Hence, the gnomic projection is particularly useful when sailing great circles (like when you dabble in circumnavigation) and is beyond the scope of a coastal navigation

course.

## Organization of the chart

- **Authority:** The publisher responsible for the information in the chart. "British Admiralty Charts" or "Imray Charts".
- **Title:** The title gives a description of the area covered by the chart. For example: "The Aegean Sea of Greece - Athens to Rhodes".
- **Number:** Different chart types of the same area can be distinguished by the chart's number.
- **Projection:** Most likely the Mercator projection as described above. Charts covering small areas can be constructed by stereographic projection.
- **Scale:** For example: 1:193000. But since the chart is distorted this holds only for one specific latitude in the Mercator chart. The scale indicates how detailed the chart is (here 1 cm on the chart represents 193000 cm on earth).
- **Horizontal geodetic datum:** The definition of the relationship between the ellipsoid adopted as the model of the Earth's shape, and the Earth itself. Though there are hundreds of datums in use, most are only locally valid.

Yet, the **WGS-84 datum** is global in scope and positions obtained by satellite navigation systems are usually referred to this datum. Therefore, a correction needs to be applied to a WGS-84 GPS position to agree with charts using other horizontal datums. For example to correct WGS-84 to the European datum, add 0.06'N, 0.04'E to the WGS-84 position indicated by the GPS. Fortunately, most GPS receivers may be set to display positions in several other datums besides WGS-84 and perform the calculations for you.

- **Chart sounding datum:** The tidal datum to which soundings and drying heights on a chart are referred. Often shortened to “chart datum” when it is clear that reference is not being made to a horizontal datum. Chart sounding datums are also used as reference for heights (lighthouses, mountains, bridges). Multiple datums can be used in one chart: L.A.T. for soundings and M.L. for heights.
- **Soundings & height units:** Soundings and heights can be stated in - for example - meters, feet or fathoms. Nowadays, even most British charts have adopted the metric system.
- **Horizontal scale:** Natural scale at for example 40° 15'.3 latitude where the horizontal scale can be used for measuring distances and where the chart scale is true.
- **GPS compatibility:** Most charts neither have the precision nor the resolution to fully use the (differential) GPS positioning potential. Moreover, still plenty of charts result from surveys done in the 19th century.
- Also, GPS data often requires a correction for a local horizontal chart datum before it can be used in the chart.
- **Corrections & edition:** The chart is for example a 2004 edition but is - when properly corrected - still valid in 2006. Corrections are published continuously and the changes should be mentioned in the bottom left corner of the chart.

## Information in the chart

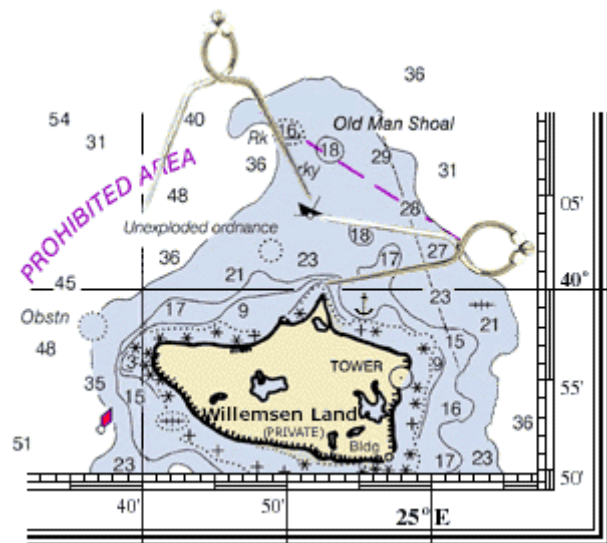
- **Depths reduced to chart datum:** A sounding like ( $3^5$ ) indicates 3½ metres of water under Lowest Astronomical Tide (when depths are notated in “metres” and the chart datum is “L.A.T.”). An underlined sounding like (0<sup>4</sup>) indicates a height of 40 cm above L.A.T..
- **Isobaths:** Lines connecting positions with the same depth: depth contours.
- **Heights reduced to chart datum:** Heights of for instance, lighthouses, mountains and cliffs are more often reduced to another datum such as Mean High Water (M.H.W.) or Mean High Water Spring.
- **Tidal information:** Details of both the horizontal and the vertical movement of the water is often included in the chart.
- **Buoys & marks:** Lightships, lateral and cardinal marks.
- **Seabed qualities:** Pebbles, seaweed, rocks, wrecks, pipelines, sand (fine or coarse) and other **seabed characteristics for anchoring**.
- **Lighthouses:** Their height, colour, range, and other properties, **see chapter 9**.
- **Magnetic variation:** The angle between the true North and the magnetic North varies in place and time. The local variation is indicated in the compass card [\[i\]](#).
- **Conspicuous positions on the shore:** Churches, radio masts, mountain tops, etc. that can be used for compass bearings and other means of navigation.

## Coordinates and positions

We use a pair of nautical dividers to obtain precise coordinates from the chart. This gadget enables you to take the distance between that particular position and the closest grid line. You then place the dividers on the scale with one end on this grid line, leaving the other end precisely at your coordinate. Do this twice to get both latitude and longitude.

Below are some examples.

To find a position on the chart is done by reversing this method.



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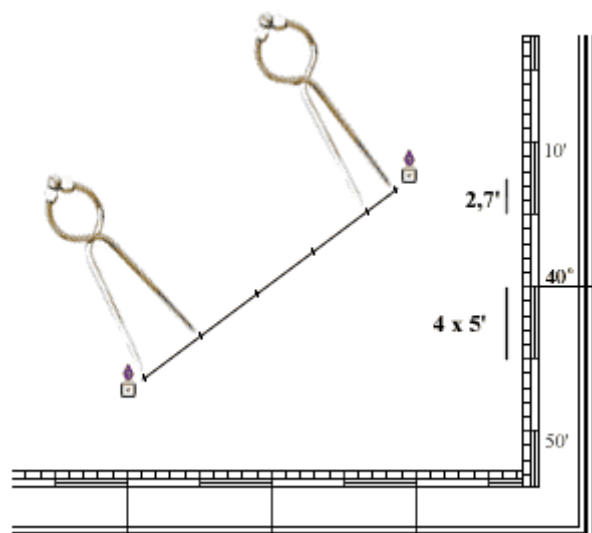
	<b>Visual wreck</b>	40° 04',8 N , 24° 52',0 E
	<b>Tower</b>	39° 55',0 N , 24° 58',0 E
	<b>Dangerous wreck</b>	39° 52',8 N , 24° 42',2 E
	<b>Anchorage</b>	39° 58',5 N , 24° 55',7 E
	<b>Buoy with red light</b>	39° 52',5 N , 24° 37',2 E

Some chart symbols come with a little circle “o” indicating their precise location, such as the “visual wreck” symbol.

## Distances

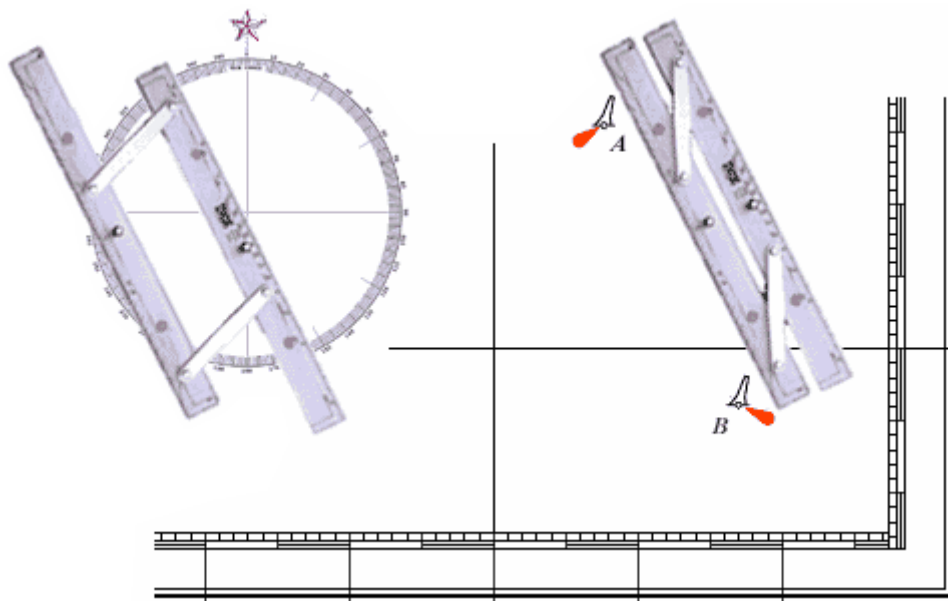
To measure the distances between, for instance, these two **oil rigs**, we will again need our dividers. Remember, we can only use the vertical scale.

We first take a 'nice' distance like 5' (5 nautical miles) on the vertical scale using the middle latitude. Then we start walking with the dividers from one oil rig to the other. Finally, we adjust the dividers to measure the small remaining part at its own height (its own latitude). Were you to measure it below 40° you would read 2.5' instead of 2.7' !



## Courses

So, now we can measure distances and both plot and read out positions, but we also need directions. For example we need to find the course from buoy **A** to buoy **B**. To accomplish this we may use parallel rules as shown in this chart.



First you line this instrument up with the two buoys on the right. Then follows the intriguing part in moving the device to the compass rose without losing its alignment. Finally, when one of the rules is aligned with the heart of the compass card, you can read course A-B. In this example: 153°.

Besides the parallel rules there are other types of instruments available.

## Selection of chart symbols



Super(light)buoy, Lanby.



Lateral green starboard hand buoy.



Safe water mark (red/white).



Stone; drying height above chart datum.



Danger, least depth by sounding.



Wreck visible at chart datum.



Dangerous wreck, depth unknown.



Position for which tidal levels are tabulated.



Sector light, colour changes on different bearings.



Oil rig, prohibited zone of 500 m.

**Obstn**

Obstruction.

**PA**

Position Approximate.

**St**

Stones - Danger.

**FL 42m 29M** Flashing light, 42 meters above datum, range 29'.



Light buoy.



Lateral red port hand buoy.



Cardinal buoy, West mark.



Foul seabed. Avoid anchoring here.



Danger, depth swept by wire drag.



**Mt(s)** Wreck showing Mast(s) above chart datum.



Wreck, not dangerous (10 m below chart datum).



Position for which tidal stream data are tabulated.



Danger line, in general.



Lighted platform, prohibited zone of 500 m.

**Wd**

Seaweed, describes seabed.

**P**

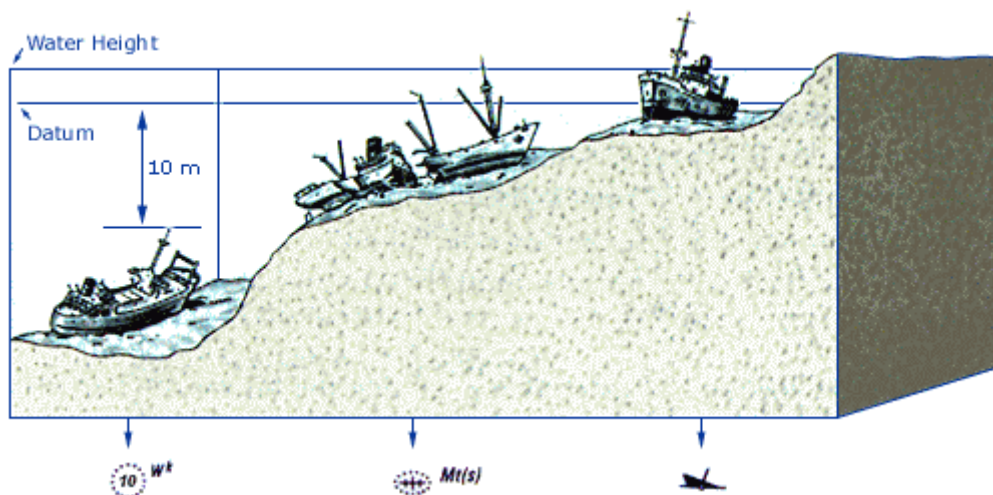
Pebbles, the seabed.

**Whis**

Whistle Buoy.

**LFI 10s**

Long flashing light, period 10 seconds.



So, now it is clear that you can find in the chart: “cardinal marks”, “chart sounding datums” and “lighthouses that *long flash* at you”, but what does all this mean?

Well, “lights” along with “Lateral and Cardinal Buoys” are dealt with in great detail in the **chapter 9 - navigation aids**, while the exact meaning of “chart datums” will be explained in **chapter 6 - tides**.

For now, just remember that all this information can be found in the nautical chart.

## Overview

**Mercator projection:** Most coastal nautical charts are constructed with this method. Angles are true and distances can be measured using the vertical scale.

**Stereographic projection:** Used for chart covering small areas. Like the Mercator projection use the vertical scale to measure distances.

**Gnomic projection:** Used for vast areas. Great circles appear as straight lines on the chart.

**Great circle navigation:** The shortest course on earth between two positions is a great circle; for circumnavigating and ocean crossings.

**Loxodrome:** A line which makes the same angle with all meridians. Theoretically not the shortest route, but a handy straight line on a Mercator chart.

**Horizontal geodetic datum:** Defines the relationship between the ellipsoid adopted as the model of the Earth's shape, and the Earth itself. Coordinates which refer to, for instance, AIA [\[i\]](#) should be corrected before plotting them in a chart based on another horizontal datum. If your GPS receiver consistently disagrees with known positions by a constant amount and direction, then check that you have set it to display the correct horizontal datum.

**Chart sounding datum:** The tidal datum (fictitious plane) to which soundings, heights, elevations and drying heights on a chart are referred.

**Vertical scale:** Distances in nautical miles or minutes (') should be measured at the same latitude on the vertical scale.

**Corrections:** Each chart is liable to corrections which are published by either a national body or the publisher of the nautical chart.

# Compass Navigation

## Marine compass

In **China** compasses have been in use since the Han dynasty (2nd century BCE to 2nd century CE) when they were referred to as “south-pointers”. However at first these magnets were only used for geomancy much like in the art of Feng Shui.

Eventually, during the Sung dynasty (1000 CE) many trading ships were then able to sail as far as Saudi Arabia using compasses for marine navigation. Between 1405 and 1433, Emperor Chu Ti's Treasure Fleet of the Dragon Throne ruled the entire South Pacific and the Indian Ocean, a territory that ranges from Korea and Japan to the Eastern coast of Africa.

At this time Western mariners were still rather ignorant of the navigational use of the magnet. Petrus Perigrinus van Maricourt [!\[\]\(8b57f0e15e7dda24cf9977561475f640\_img.jpg\)](#) wrote a first treatise on the magnet itself: “De Magnete” (1269). And though its nautical use was already mentioned in 1187 by the English monk Alexander Neckham, the use onboard only came about around the 13th and 14th century in the Mediterranean Sea.

Much later, in 1545, Pedro de Medina (Sevilla 1493-1567) wrote the Spanish standard work “Arte de Navegar” on marine compass navigation. This masterpiece was first translated in Dutch (1580) and was -O Irony- used by Jacob van Heemskerck [!\[\]\(4cafc60cd39da821525d7c6589540296\_img.jpg\)](#) when the Dutch destroyed the Spanish fleet near Gibraltar in 1607. The drawback was of course Van Heemskerck's own death during this victory.

## Magnetic Variation

In the fin-de-siècle of the sixteenth century mariners believed that the magnetic north pole coincided with the geographic north pole. Any suggestion otherwise had been denied by Pedro de Medina.

Magnetic observations made by explorers in subsequent decades showed however that these suggestions were true. But it took until the early nineteenth century, to pinpoint the magnetic north pole somewhere in Arctic Canada (78° N , 104° W). From then on the angle between the true North and the **Magnetic North** could be precisely corrected for. This correction angle is called **magnetic variation** or declination.

It is believed that the Earth's magnetic field is produced by electrical currents that originate in the hot, liquid, outer core of the rotating Earth. The flow of electric currents in this core is continually changing, so the magnetic field produced by those currents also changes. This means that at the surface of the Earth, both the strength and direction of the magnetic field will vary over the years. This gradual change is called the **secular variation** of the magnetic field. Therefore, variation changes not only with the location of a vessel on the earth but also varies in time.

## Correcting for variation

The correction for magnetic variation is shown for your location on your current navigation chart's compass rose. Take for example a variation of 2° 50' E in 1998. In 2000, this variation is estimated to be 2° 54', almost 3° East. This means that if we sail 90° on the chart (your true course), the compass would read 87°.

To convert your **true course** into a **compass course** we need first assign a “-” to a Western and a “+” to a Eastern variation. From the following equation you will see that this makes

sense [i](#):

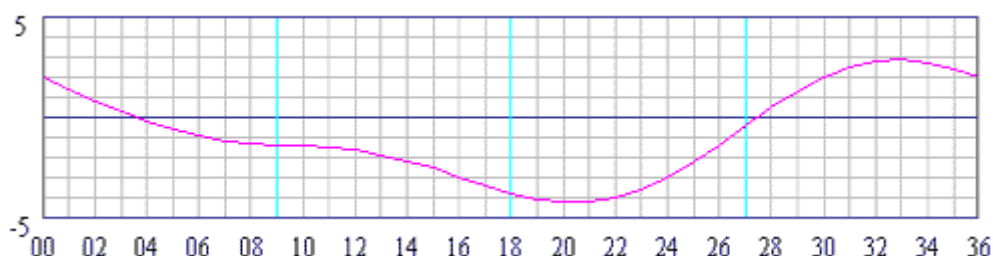
$87^\circ \text{ cc} + 3^\circ \text{ var} = 90^\circ \text{ tc}$ , in which “cc” and “tc” stand for “compass course” and “true course”, respectively.

We can use the same equation to convert a compass course into a true course. If we steered a compass course of  $225^\circ$  for a while, we have to plot this as a true course of  $228^\circ$  in the chart.

## Magnetic deviation

Magnetic deviation is the second correctable error. The deviation error is caused by magnetic forces within your particular boat. Pieces of metal, such as an engine or an anchor, can cause magnetic forces. And also stereo and other electric equipment or wiring, if too close to the compass, introduce large errors in compass heading.

Furthermore, the deviation changes with the ship's heading, resulting in a deviation table as shown below. The vertical axis states the correction in degrees West or East, where East is positive.



The horizontal axis states the ship's heading in degrees divided by ten. Thus, when you sail a compass course of  $220^\circ$ , the deviation is  $4^\circ \text{ W}$ .

When a compass is newly installed it often shows larger deviations than this and needs compensation by carefully placing small magnets around the compass. It is the remaining error that is shown in your deviation table.

You can check your table every now and then by placing your boat in the line of a pair of **leading lights** and turning her 360 degrees.

## Correcting for both deviation and variation

Converting a compass course into a true course, we can still use our equation but we need to add the correction for deviation:

$$\text{cc} + \text{var} + \text{dev} = \text{tc}$$

- **Example 1:** The compass course is  $330^\circ$ , the deviation is  $+3^\circ$  (table) and the variation is  $+3^\circ$  (chart);

$$330^\circ \text{ cc} + 3^\circ \text{ var} + 3^\circ \text{ dev} = ?^\circ \text{ tc}$$

giving a true course of  $336^\circ$  which we can plot in our chart

- **Example 2:** The compass course is  $220^\circ$ , the deviation is  $-4^\circ$  (table) and the variation is still  $+3^\circ$  (chart).

$$220^\circ \text{ cc} + 3^\circ \text{ var} + -4^\circ \text{ dev} = ?^\circ \text{ tc}$$

- giving a true course of  $219^\circ$  which we can plot in our chart.

Converting a true course into a compass course is a little less straight forward, but it is still done with the same equation.

- **Example 3:** The true course from the chart is  $305^\circ$  and the variation is  $+3^\circ$  (chart), yet we don't know the deviation;

$$?^{\circ} \text{ cc} + 3^{\circ} \text{ var} + ?^{\circ} \text{ dev} = 305^{\circ} \text{ tc}$$

Luckily, rewritten this reads:

$$305^{\circ} \text{ tc} - + 3^{\circ} \text{ var} = \text{cc} + \text{dev} = 302^{\circ}$$

In plain English: the difference between the true course and the variation ( $305 - + 3$ ) = 302 should also be the summation of the compass course and the deviation. So, we can tell our helms person to steer  $300^{\circ}$ , since with a cc of  $300^{\circ}$  we have a deviation of  $+2^{\circ}$  (As can be deduced from the deviation table above).

- **Example 4:** The true course from the chart is  $150^{\circ}$  and we have a Western variation of 7 degrees ( $-7^{\circ}$ ). We will use the rewritten equation to get:

$$150^{\circ} \text{ tc} - - 7^{\circ} \text{ var} = \text{cc} + \text{dev} = 157^{\circ}$$

From the deviation table we find a compass course of  $160^{\circ}$  with a deviation of  $-3^{\circ}$ .  
Voilà!

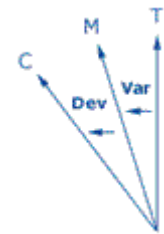
## Magnetic course

The **magnetic course** (mc) is the heading after magnetic variation has been considered, but without compensation for magnetic deviation. This means that we are dealing with the rewritten equation from above:

$$\text{tc} - \text{var} = \text{cc} + \text{dev} = \text{mc}$$

Magnetic courses are used for two reasons. Firstly, the magnetic course is used to convert a true course into a compass course like we saw in the last paragraph. Secondly, on boats with more than one compass more deviation table are in use; hence only a magnetic or true course is plotted in the chart.

To summarise, we have three types of “north” (true, magnetic and compass north) like we have three types of courses (tc, mc and cc). All these are related by deviation and variation.



## Overview

**Variation:** The angle between the magnetic north pole and the geographic north pole. Also called the magnetic declination [Li](#).

**Secular variation:** The change of magnetic declination in time with respect to both strength and direction of its magnetic field.

**West (-) , East (+):** Western variations or deviations are designated with a negative sign by convention due to the compass card's clock wise direction.

**Deviation:** The error in compass heading caused by electric magnetic currents and or metal objects.

**Deviation table:** A table containing deviations in degrees versus the ship's heading (compass course) in degrees. Usually plotted in a graph.

**True course:** Course plotted in the chart i.e. course over the ground or “course made good”. The course corrected for compass errors.

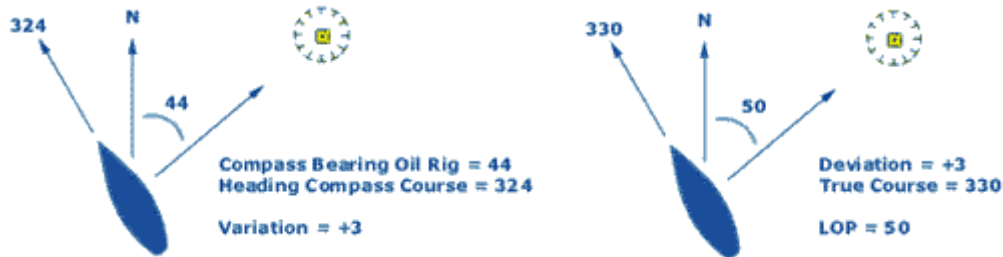
**Compass course:** The course (ship's heading) without the correction for compass errors.

**cc + var + dev = tc:** This equation shows the connection between the compass course, its errors and the true course. It can also be read as:  $\text{tc} - \text{var} = \text{cc} + \text{dev}$ .

# Plotting and piloting

## Lines of Position

The modern chart shows us positions of many recognizable navigation aids like churches and lighthouses, which facilitate the approach to a coastal area. This concept originated from a chart by Waghenaeer [\[i\]](#) and proved a milestone in the development of European cartography. This work was called “Spiegel der Zeevaerdt” [\[i\]](#) and included coastal profiles and tidal information much like the modern chart. It enables us to find the angle between the North and for example a platform, as seen from our position.



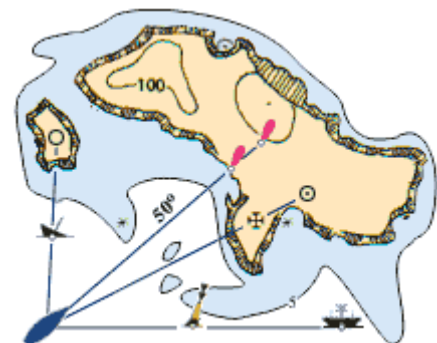
Taking a bearing on this oilrig with a compass provides us with a compass course. This course first needs correction for both variation and - via ship's heading - **deviation** [\[i\]](#) before plotting a **Line of Position** (LOP) in the chart. Our position is somewhere along this line.

## Ranges

A precise way to obtain a LOP (and without a compass) is to locate two navigational aids in line. The image on the right shows us four examples of **ranges**, each consisting of two nav. aids.

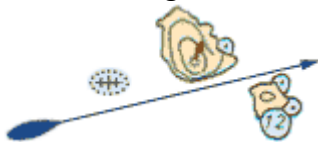
Please, note that:

- More distance between the two nav. aids enhances accuracy.
- And less distance between the vessel and the closest nav. aid also enhances accuracy.



One of the four ranges consists of two lights that are intentionally placed to provide a LOP. These pairs of lights are called **Range lights** or **Leading lights**. In this case they indicate the channel between the shallows along a true course of 50° [\[i\]](#).

When looking toward the leading lights, the closest one will be lower [\[i\]](#).

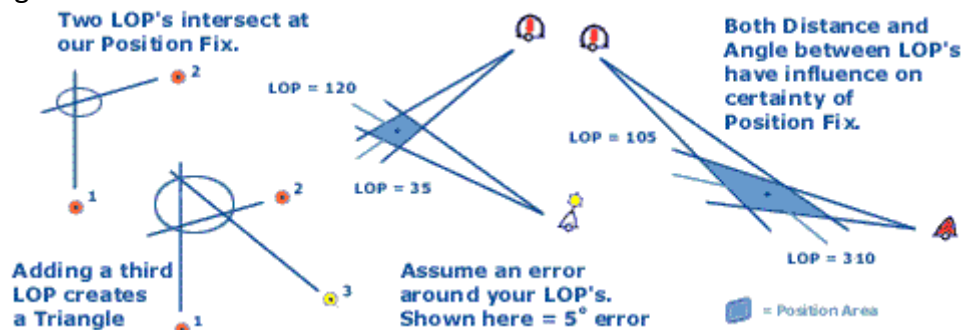


Therefore, in the middle of the channel both lights will appear above each other. Even when there are no man-made structures available, a range can be found by using natural features such as coastlines and islets. The example on the left shows a yacht that will avoid the dangerous wreck as long as the islets don't overlap.

## The Position Fix

To construct our position fix we need two of these lines of position to intersect each other. **Fix** [\[i\]](#) is the initial element of the ship's navigational and **dead reckoning** (see below) plot. A fix is the ship's position on the earth at some given point in time. A fix is determined by the

simultaneous intersection of LOP's. Often however, a triangle occurs when a third LOP is added in the construction. This indicates that there are errors involved in at least one of the bearings taken. In practice, we should consider each LOP as the average bearing in a wider sector of, for instance  $10^\circ$ . Bearings create more certainty about our position when they are perpendicular to each other. Yet, bearings on distant objects bring about more uncertainty in our position fix as the sector widens. If moving fast you should not put any time between the bearings.

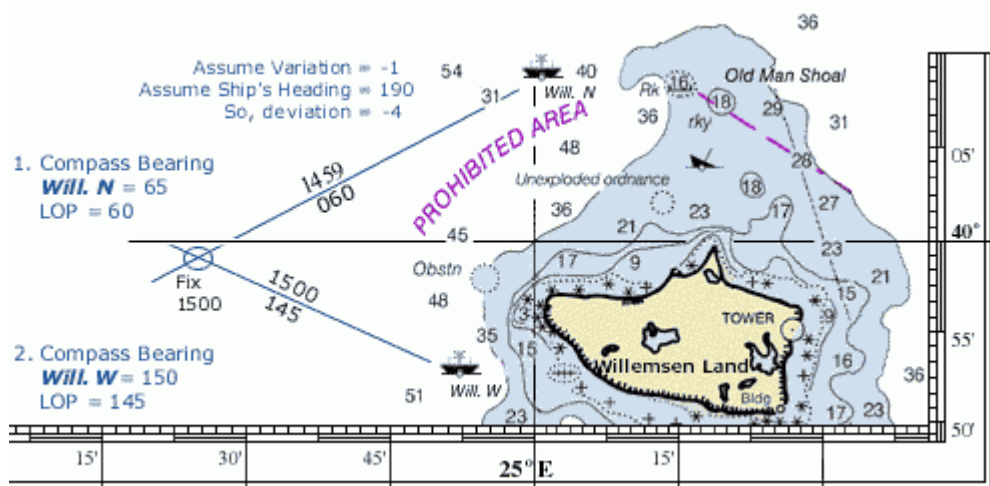


In the next example we will plot our position fix by taking bearings at two light vessels just off the coast of Willemssen Land. To plot in the chart we will use a soft pencil and avoid drawing lines through the chart symbols. This is to prevent damage to the chart when you have to erase the construction.

Since we use our steering compass for our bearings the same deviation table can be used. We will assume the variation to be  $-1^\circ$  and the ship's compass heading  $190^\circ$ . Hence, from the deviation table we find a deviation of  $-4^\circ$ .

#### The construction:

- The first compass bearing on 'Will. N' is  $65^\circ$ .  
cc + var. + dev. = tc, therefore tc =  $60^\circ$ .
- Plot the LOP in the chart aligned to this lightship. Mark 'Time' and 'True Course' along with it.
- Mutatis mutandis [\[1\]](#) the second LOP on 'Will. W' is  $145^\circ$ .
- The intersection of these two LOP's is our Position Fix. Mark this with an 'Ellipse' and the 'Time'. The greater the uncertainty, the greater the ellipse (position area).
- Fixed Position around  $15^h00^m = 39^\circ 58'.9 \text{ N}$ ,  $24^\circ 25'.5 \text{ E}$ , approximately. So, welcome to my island but mind the rocks!

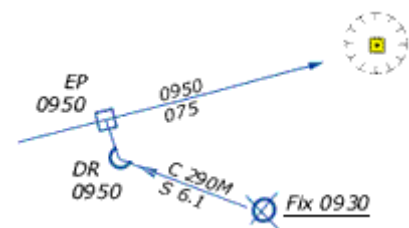


Although we didn't have a third LOP creating the dreaded triangle, we still have to doubt the accuracy of our position fix. Then, if three or more LOP's were used, and a nice point was not achieved, we again are left with some ambiguity. This could be caused by any number of reasons, including instrument errors, erroneous identification of a navigation aid, sloppy plotting, or error by the bearer taker, among others. In this case, we will assume that we are at the worst possible position (i.e. closest to the nearest navigational hazard). To minimize the effect of possible errors the **optimum angular spread** should be **90°** when two objects are shot or **120°** when three objects are shot.

## The Estimated Position

It is sometimes impossible to obtain more than one LOP at a time. To determine the ship's position using only one navigation aid, we can use a **running fix** (see below). However if a running fix is not possible, we can determine an estimated position.

An estimated position is based upon whatever incomplete navigational information is available, such as a single LOP, a series of depth measurements correlated to charted depths, or a visual observation of the surroundings. An estimated position can be determined using a single LOP and the ship's **Dead Reckoning Position** (DR) [Li](#). This is done by drawing a line from the DR position at the time of the LOP perpendicular to the LOP. An EP is denoted by a square instead of an ellipse used for a fix.



Do not rely on an EP as much as a fix. The **scale of reliability**, from best to worst:

- Fix
- Running Fix
- Estimated Position
- DR position

## Dead Reckoning

Dead reckoning [Li](#) is a technique to determine a ship's approximate position by applying to the last established charted position a vector or series of vectors representing true courses and speed. This means that if we have an earlier fix, we plot from that position our course and 'distance travelled since then' and deduce our current position.

**0930: We start off with a Fix and plot a DR position for 15 minutes later.**

**0945: Our estimation about our speed and course was correct, so we don't have to change the DR position.**

**1000: and so on...**

**S = Speed through water**

**C = Course (T = true, M = magnetic, C = compass) through water.**

**Mark with an arrow as indicated.**



Dead reckoning is crucial since it can provide an approximate position in the future. Each time a fix or running fix is plotted, a vector representing the ordered course and speed originate from it. The direction of this **course line** represents the ship's true course, and the length represents the distance one would expect the ship to travel in a given time. This

extrapolation is used as a safety precaution: a predicted DR position that will place the ship in water 1 meter deep should raise an eyebrow...

#### Guide-lines:

- Plot a new course line from each new fix or running fix.
- Never draw a new course line from an EP.
- Plot a DR position every time course or speed changes.
- Also, plot a DR position when a single LOP is obtained.
- Label the DR position with a semi-circle and 'DR'.

### Running Fix

Under some circumstances, such as low visibility, only one line of position can be obtained at a time. In this event, a line of position obtained at an earlier time may be advanced to the time of the later LOP. These two LOP's should not be parallel to each other; remember that the optimal angular spread is 90°. The position obtained is termed a running fix because the ship has "run" a certain distance during the time interval between the two LOP's.

**0905:**

**Our tacking sequence starts with a solid position Fix.**

**0916:**

**We obtain a single LOP and construct a corresponding (same time) dead reckoning position. Our estimated position is constructed by drawing the shortest line between the DR and the LOP (perpendicular).**

**0926:**

**No LOP's at all.**

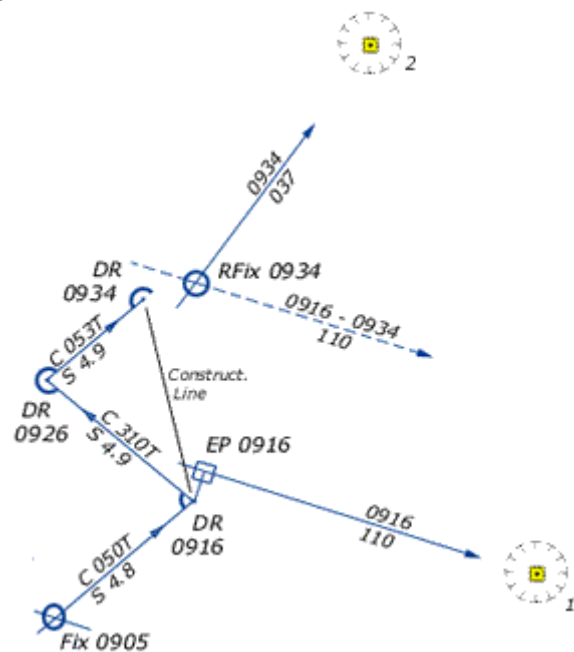
**We tack and construct a DR position.**

**0934:**

**We obtain a LOP on Oil Rig 2.**

**To use the first LOP we advance it over a construction line between the two corresponding DR positions.**

**We use both its direction and distance.**



To use the LOP obtained at the earlier time, we must advance it to the time of the second LOP. This is done by using the dead reckoning plot. First, we measure the distance between the two DR positions and draw a **construction line**, which is parallel to a line connecting the two DR positions.

Note that if there are no intervening course changes between the two DR positions, it's easiest just to use the course line itself as the construction line.

Now, using the parallel rulers we advance the first LOP along this construction line over the distance we measured. Et Voilà, the intersection is our RFix.

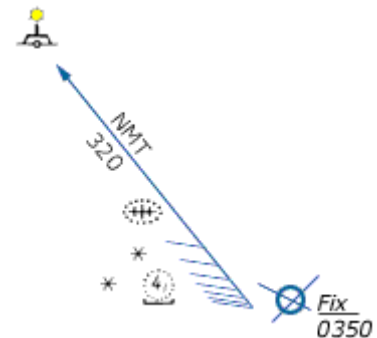
If there is an intervening course change, it appears to make our problem harder. Not so! The only DR positions that matter are the two corresponding with the LOP's.

### Advancing a LOP:

- The distance: equal to the distance between the two DR positions.
- The direction: equal to the direction from the first to the second DR position.
- Label the Running Fix with an ellipse and "RFix".

## Danger Bearing

Like the dead reckoning positioning the danger bearing is an important tool to keep the ship out of trouble. First, the navigator identifies the limits of safe, navigable water and determines a bearing to a prominent landmark. This bearing is marked as "No More Than" (**NMT**) or "No Less Than" (**NLT**), depending on which side is safe. Hatching is included on the side that is hazardous, along with its compass bearing. When a distance instead of a direction is used a **danger range** is plotted much the same way as the danger bearing.



## Turn Bearing

The Turn bearing - like the danger bearing - is constructed in the chart in advance. It should be used as a means of anticipation for sailing out of safe waters (again like the danger bearing and dead reckoning). The turn bearing is taken on an appropriate navaid and is marked 'TB'. As you pass the object its bearing will slowly change. When it reaches the Turn bearing turn the vessel on her new course.

This type of bearing is also used for selecting an anchorage position or diving position.

## Snellius Construction

Willebrord Snellius [\[i\]](#) - a 16th century mathematician from Leiden NED - became famous for inventing the loxodrome and his method of triangulation.

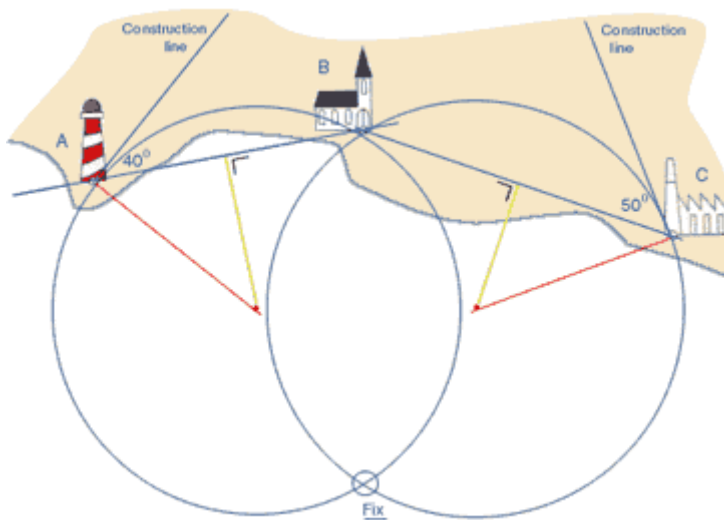
The **Snellius construction** was first used to obtain the length of the meridian by measuring the distance between two Dutch cities [\[i\]](#). He took angles from and to church towers of villages in between to reach his objective. We now can use the Snellius method to derive our position from three bearings without the use of LOP's. We can leave out deviation and variation, which simplifies things. Also, since only relative angles are needed a sextant can be used to measure navigation aids at greater distances.

### The construction:

- Compass bearings are  $320^\circ$  on A,  $360^\circ$  on B and  $050^\circ$  on object C.
- The angle between A and B =  $40^\circ$ .
- The angle between B and C =  $50^\circ$ .
- Draw lines from A to B and from B to C.
- Add the two perpendicular bisectors (yellow) of lines AB and BC.
- Draw at object A a construction line (blue)  $40^\circ$  inland of line AB.
- Draw at object C a second construction line (blue)  $50^\circ$  inland of line CB.



- At object A: draw a new line (red) perpendicular to the construction line.
- At object C: draw another new line (red) perpendicular to the construction line.
- The two intersections of the red and yellow lines indicate the centres of two circles.
- Finally, draw the first circle using A and B and the second circle using B and C.
- The off shore intersection of the two circle gives us our position fix.



The advantage: deviation and variation can be left out since the angles (here 40° and 50°) are relative ones. Moreover, a sextant can be used to obtain angles between objects at greater distances, which with a compass would be less precise.

## Notation

Fix:		<u>Fix 1530</u>
Running Fix:		<u>RFix 0911</u>
Estimated Position:		<u>EP 2311</u>
Dead Reckoning:		<u>DR 1747</u>
Electronic Fix (GPS):		<u>GFix 1903</u>
Electronic Fix (Radar):		<u>RaFix 1112</u>

LOP		<u>1159</u> <u>310</u>
LOP advanced		<u>0900 - 0917</u> <u>050</u>
Course/Speed		<u>C 285C</u> <u>S 14.2</u>
Set/Drift		<u>SET 210</u> <u>DFT 0.9</u>

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## Overview

**Line Of Position (LOP):** The locus of points along which a ship's position must lie. A minimum of two LOP's are necessary to establish a fix. It is standard practice to use at least three LOP's when obtaining a fix, to guard against the possibility of and, in some cases, remove ambiguity.

**Transit fix:** The method of lining up charted objects to obtain an LOP.

**Leading lights** or **Range lights:** A pair of lights or day marks deliberately placed to mark a narrow channel.

**Position fix:** The intersection of various LOP's.

**Cross bearing:** The use of LOP's of several navigational aids to obtain a position fix. Remember to use an optimal angular spread.

**Running fix:** The use of an advanced LOP. Make sure to use only the corresponding DR positions. Also don't use the EP for advancing the first LOP.

**Dead reckoning:** Determining a position by plotting courses and speeds from a known position. It is also used to predict when lights become visible or to determine the set and rate of a current.

**Estimated position:** Combine a corresponding DR position with a single LOP to get an EP position.

**Snellius construction:** Another way to combine three compass bearings to obtain a position fix. The advantage over a cross bearing is that both magnetic variation and deviation don't need to be taken into account.

**Course:** (C) The direction in which a vessel is steered or is intended to be steered (direction through the water).

**Speed:** (S) The speed of the boat through the water.

**Set:** (SET) The direction in which the current is flowing (see chapters 6,7 and 8).

**Drift:** (DFT) The speed (in knots) of the current (see chapters 6,7 and 8).

**Default heading** is True course (M = magnetic , C = compass).

**Default time** is 24 hour clock ship time else UTC.

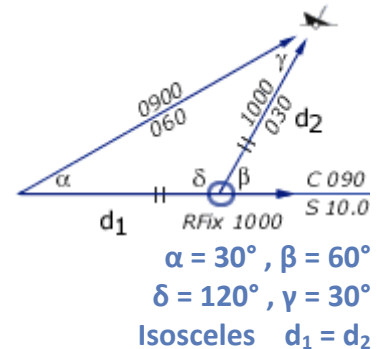
# Piloting and navigation

## Doubled angle fix

The **Doubled angle on the bow fix** resembles a running fix though only one navigation aid is used.

In the example on the right the initial angle ( $30^\circ$ ) on the bow is doubled ( $60^\circ$ ) yielding an isosceles triangle [\[i\]](#). The distance travelled between the bearings is the same as the distance from the visible wreck.

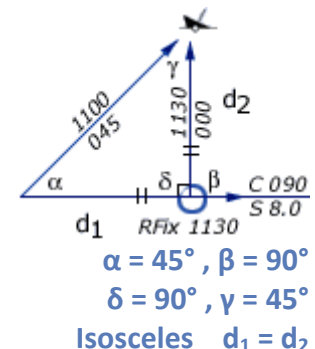
- Start with the visible wreck having a bearing of less than  $45^\circ$  off the bow ( $\alpha$ ), note the log distance.
- Proceed along the course until the angle on the bow is doubled ( $\beta$ ), read the log:  $d_1$  is 10 nm.
- Use the log distance to find the position on the second LOP. It is an isosceles triangle, so  $d_2$  is also 10 nm.
- Label it with an ellipse and "RFix" but realize it is less precise than a running fix that involves two navigation aids.



## Four point fix

If the first angle on the bow is  $45^\circ$ , a special situation occurs: The **Four point fix**, so called since 45 degrees equals 4 points on the compass (1 point =  $11,25^\circ$  [\[i\]](#)).

- Start with a bearing with  $45^\circ$  on the bow ( $\alpha$ ), note the log.
- Proceed along the course till the angle on the bow is  $90^\circ$  ( $\beta$ ), read the log:  $d_1$  is 4 nm
- Use the log distance to find the position on the second LOP. Isosceles, so  $d_2$  is also 4 nm.
- Label it with an ellipse and "RFix".



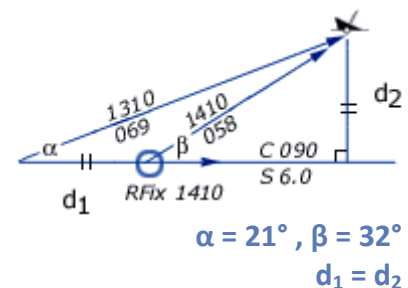
## Special angle fix

The **Special angle fix** requires the mariner to know some special pairs of angles (a : b) that give the distance travelled between bearings as equal to the distance abeam [\[i\]](#).

In the example on the right  $\alpha = 21^\circ$  and  $\beta = 32^\circ$  are used. Now, the log distance equals the shortest distance between wreck and course line (6 nm).

A few practical pairs:

16 : 22	21 : 32
25 : 41	32 : 59
37 : 72	40 : 79



Remember: the greater the angular spread the better. Hence, of these three fixes the four point fix is the most precise one.

Enter  $\alpha$  (1-45°):

$\beta$ :

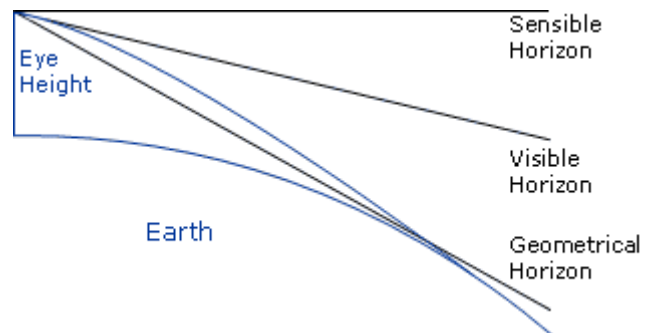
Mathematics: isosceles triangle fixes

## Distance of the horizon

On a flat world there would be no difference between the visible and sensible horizon. However, on Earth the **visible horizon** appears several arc minutes below the **sensible horizon** due to two opposing effects:

- the curvature of the earth's surface;
- atmospheric refraction.

Atmospheric refraction bends light rays passing along the earth's surface toward the earth. Therefore, the **geometrical horizon** [i](#) appears elevated, forming the visible



horizon.

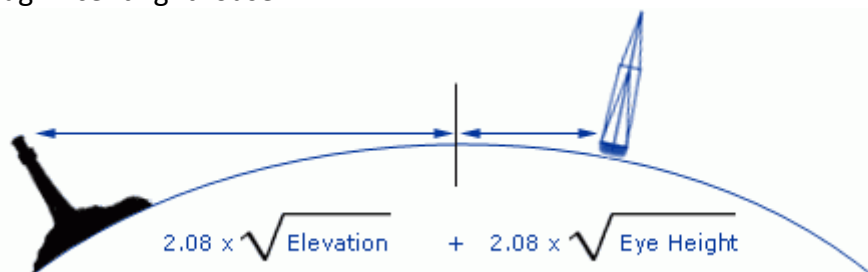
The distance of the visible horizon is a (semi-empirical) function of Eye Height:

$$2.08 \times \sqrt{\text{Eye Height}} \quad \text{a}$$

**Mathematics: horizon distances**

## Dipping range

If an object is observed to be just rising above or just dipping below the visible horizon, its distance can be readily calculated using a simple formula. The object's **elevation** (the height of a light above chart datum [i](#)) can be found in the chart or other nautical publication such as the 'List of Lights'. Note that in some charts elevation is referred to a different datum than soundings [i](#). Click on the image on the right to view a magnificent lighthouse.



The formula contains the two distances from the visible horizon and can be simplified by the equation:  $2.08 \times (\sqrt{\text{Elevation}} + \sqrt{\text{Eye height}})$  [i](#). Many nautical publications contain a table called "distances of the horizon" which can be used instead of the equation. Use the dipping range to plot a **Distance LOP** [i](#) in the chart: a circle equal in radius to the measured distance, which is plotted about the navigation aid. Finally, take a bearing on the object to get a second LOP and a position fix.

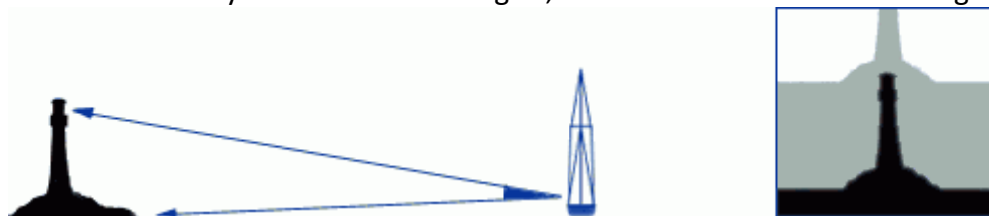
Enter Eye height (metres):

Enter Elevation (metres):

Distance is (nm):


## Vertical sextant angle

Similarly, a distance LOP can be obtained by using a sextant to measure the angle (arc) between for instance the light [\[i\]](#) and chart datum of a lighthouse or any other structure of known elevation. Once the angle is corrected for **index error** the distance can be found in a table called: "Distances by Vertical Sextant Angle", which is based on the following equation.



$$\text{Range} = 1.856 \times \frac{\text{Elevation} \pm \text{Water Height}}{\text{Angle}}$$

- The angle in minutes total, thus  $1^\circ 12' = 72'$  total, and corrected for index error.
- Elevation in metres [\[i\]](#).
- Water height in metres above or below chart datum of object.
- Distance or Range in nautical miles.
- Ascertain whether the base of the object is beyond the horizon
- Corrected angle should be greater than  $20'$ .

Though tables can be used for quick reference, this function is valid for objects higher than usually tabulated [\[i\]](#). An example with a lighthouse of 80 metres:

- Measured angle is  $1^\circ 19'$ , index error is  $+6'$ : angle =  $73'$ .
- Let's assume water height at 3 metres above Mean Level datum.
- Range =  $1.854 \times (80 - 3 / 73) = 1.96$  nm.

The range can be used as a danger bearing.

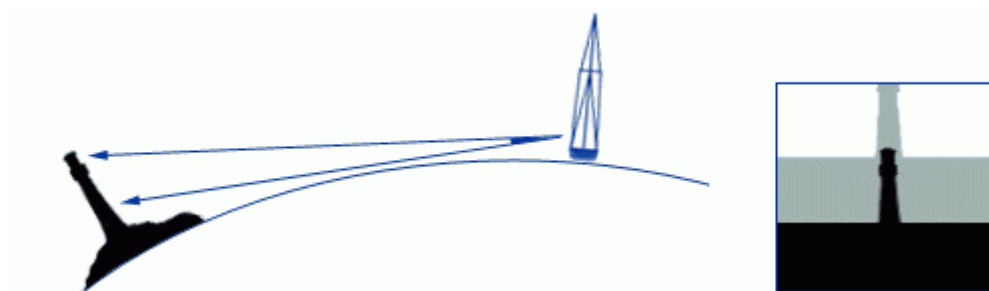
Together with a compass bearing one object with known elevation results in a position fix. If more than one vertical sextant angle is combined the optimum angular spread should be maintained.

Enter Angle (minutes total [\[i\]](#)):

Enter Elevation (metres):

Distance is (nm):

Often, the correction for water height can be left out. Though, realizing that the horizon is closer than one might think [\[i\]](#), another correction is sometimes needed. In the Mediterranean Sea for example we can see mountain tops with bases lying well beyond the horizon. Mutatis mutandis, the structures, which they bear have bases beyond the horizon as well.



$$\text{Range} = 1.854 \times \frac{\text{Elevation}}{\text{Angle} - (1.76 \times \sqrt{\text{Eye Height}})}$$

This is the equation for finding the distance of an object of known elevation located beyond the horizon. In the denominator of this equation a compensating factor is included by which the measured angle should be reduced.

Enter Eye Height (metres):

Enter Angle (minutes total [i](#)):

Enter Elevation (metres):

Distance is (nm):

### Mathematics: vertical sextant angles

#### Estimation of distance

The most obvious way to estimate distances is of course by using the distance between our eyes. If we sight over our thumb first with one eye then with the other, the thumb moves across the background, perhaps first crossing a tower second crossing a bridge.



The chart might tell that these structures are 300 m apart. Use the ratio of: distance between eye and outstretched arm/distance between pupils: usually **10** [i](#).

The objects are 3 kilometres away.

Other physical relationships are useful for quick reference. For example, one finger width held at arm's length covers about 2° arc, measured horizontally or vertically.

Two fingers cover 4°. Three fingers cover 6° and give rise to the **three finger rule**: "An object that is three fingers high is about 10 times as far away as it is high."

#### Estimation with horizon

The image on the right shows us that it is possible to estimate the height of any object that crosses the horizon as seen from our own point of view.

This picture of the 'Pigeon Rocks' near Beirut harbour was taken from a crow's nest at a height of 34 metres.

The distance of the visible horizon (12 nm) is far larger than 34 metres [i](#). Therefore, we can - without any other information - estimate that these rocks have a height of 34 metres as well.

**Factum: All tops crossing the horizon and with bases at sea level are on eye level [i](#).**

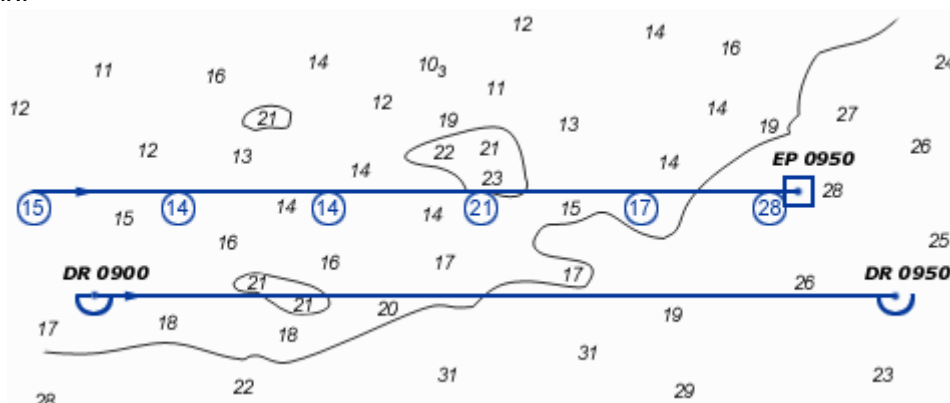


Furthermore, if we see these rocks over a vertical angle of for example  $7^\circ$   = 0.1225 rad., then the range is  $34/0.1225 = 277$  metres.

Finally, plot both range and bearing in the chart to construct an EP, et Voilà!

## Fix by depth soundings

A series of depth soundings - in this example every 10 minutes - can greatly improve your position fix:



correct your soundings for tide, etc. .

copy the DR course line on a transparent sheet;

write the depths adjacent according to the times of the soundings;

move the sheet over the chart to find its best location.

Due to leeway, currents or other factors the two course lines need not be parallel to or of same length as each other.

## Overview

**Line Of Position (LOP):** The locus of points along which a ship's position must lie. A minimum of two LOP's are necessary to establish a fix. It is standard practice to use at least three LOP's when obtaining a fix, to guard against the possibility of and, in some cases, remove ambiguity.

**Range or Distance LOP:** Obtained by using a stadimeter, sextant or radar. A circle equal in radius to the measured distance is plotted about the navigation aid; the ship must be somewhere on this circle.

**Running fix:** A position determined by crossing lines of position obtained at different times and advanced or retired to a common time.

**Dead reckoning:** Determining a position by plotting courses and speeds from a known position. It is also used to predict when lights become visible or to determine the set and drift of a current. DR positions are drawn in advance to prevent sailing into danger. A DR position will be plotted:

- every hour on the hour;
- at the time of every course change or speed change;
- for the time at which a (running) fix is obtained, also a new course line will be plotted;
- for the time at which a single LOP is obtained;
- and never draw a new course line from an EP position!

**Estimated position:** The most probable position of a craft determined from incomplete data or data of questionable accuracy. Such a position might be determined by applying a

correction to the dead reckoning position, as for estimated current; by plotting a line of soundings; or by plotting a LOP of questionable accuracy.

**Double angle on the bow:** A method of obtaining a running fix by measuring the distance a vessel travels on a steady course while the relative bearing (right or left) of a fixed object doubles. The distance from the object at the time of the second bearing is equal to the run between bearings, neglecting drift.

**Four point fix:** A special case of doubling the angle on the bow, in which the first bearing is 45° right or left of the bow. Due to angular spread this is the most precise isosceles fix.

**Special angle fix:** A construction using special pairs of relative angles that give the distance travelled between bearings as equal to the navigation aids' range abeam.

**Distance from horizon:** The distance measured along the line of sight from a position above the surface of the earth to the visible horizon.

**Sensible horizon:** The circle of the celestial sphere formed by the intersection of the celestial sphere and a plane through the eye of the observer, and perpendicular to the zenith-nadir line.

**Visible horizon:** The line where Earth and sky appear to meet. If there were no terrestrial refraction, visible and geometrical horizons would coincide. Also called : apparent horizon.

**Geometrical horizon:** Originally, the celestial horizon; now more commonly the intersection of the celestial sphere and an infinite number of straight lines tangent to the earth's surface and radiating from the eye of the observer.

**Dipping range** or **Geographic range:** The maximum distance at which the curvature of the earth and terrestrial refraction permit an aid to navigation to be seen from a particular height of eye (without regard to the luminous intensity of the light).

**Elevation:** The height of the light above its chart datum in contrast to the height of the structure itself.

**Chart Datum:** Officially: Chart Sounding Datum: An arbitrary reference plane to which both heights of tides and water depths are expressed on a chart. In the same chart heights can be related to other datums than depths.

**Vertical sextant angle:** The method of using the subtended angle of a vertical object to find its range.

**Index error:** In a marine sextant the index error is primarily due to lack of parallelism of the index mirror and the horizon glass at zero reading. A positive index error is subtracted and a negative index error is added.

**Estimation with horizon:** Estimation of heights using the horizon: All tops crossing the horizon and with bases at sea level are on eye level.

**Estimation with depth effect:** .



**Estimated position with soundings:** .


# Tides

## Tidal movements

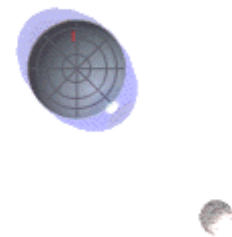
The **tide** is the vertical rise and fall of the sea level surface caused primarily by the change in gravitational attraction of the moon, and to a lesser extent the sun. As the earth spins on its axis the centrifugal force results in slightly deeper water near the equator as opposed to shallower water at the poles. In fact it causes a flow from the poles to the equator.


The earth is also in orbit around the sun (one revolution in one year) creating not only another centrifugal force but also a gravitational interaction. These two yield a bulge on the night side (centrifugal) and a bulge on the day side (gravitational) both of them moving as the world turns. Therefore, a certain place on this world will experience two high and two low tides each day.

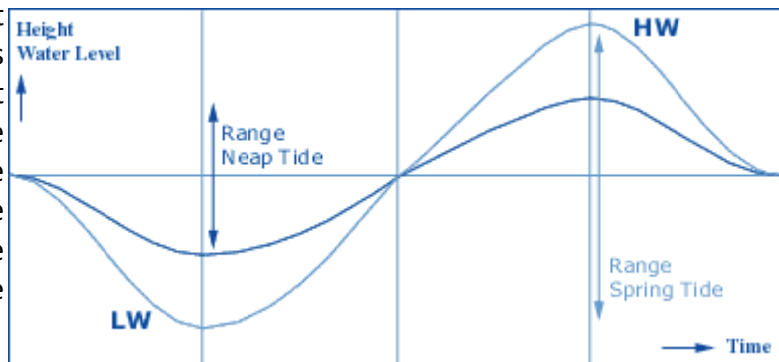
With these forces alone, we would not have **spring tides**  and **neap tides** . Spring tides have higher high tides and lower low tides whereas neap tides have lower high tides and higher low tides. Hence, the **range** (difference in water level between high and low tide) is much larger in a spring tide than in a low tide.

These differences in range can be explained if we include the moon into our earth-sun system. The moon and the earth orbit each other around a point (called the barycenter or baricenter) 2000 odd kilometres inside the earth, creating a centrifugal  and a gravitational bulge. Moreover, despite the sun's immensely larger mass, the moon exerts a 2.25 times larger gravitational attraction, since the moon is much closer to our earth.

It is the combined effect of the sun and moon that creates spring and neap tides. In the animation the gravitational forces of both the sun and the moon are taken into account. When aligned with the earth they combine their attraction and otherwise they counteract their attraction. The sun is located in the corner right below, far outside this picture (note the eclipse) while the moon is revolving round the earth. One full circle corresponds to one lunar cycle (about 28 days).



This animation shows how the tide changes during the lunar cycle. When the sun, moon and earth are aligned : **spring** tide. When at right angles the forces are not aligned: **neap** tide. The time between spring and neap is approximately 7 days.



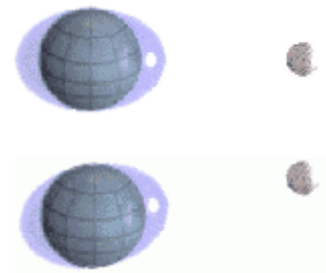
The figure below shows the ideal sinusoids of both spring and neap tides. Vertically the water height is shown versus horizontally the time. Ideally, the time between a low and a successive high is somewhat more than 6 hours.

The time difference between spring tide and neap tide is normally 7 days and is in accordance with the **phases of the moon**. Yet, water has mass and therefore momentum. Moreover, it is a viscous fluid that generates friction if moved. Therefore, the actual spring tide lags a day or so behind a full moon or new moon occurrence.

So, tidal movements are intrinsically periodical, resulting in a **Tidal day** of 24 hours and 50 minutes containing one **tidal cycle**, namely two highs and two lows. This basic pattern may be distorted by the effects of landmasses, constrained waterways, friction, the Coriolis effect, or other factors. Hence, predictions are possible and we expect the the next day's high tide to come about 50 minutes later.

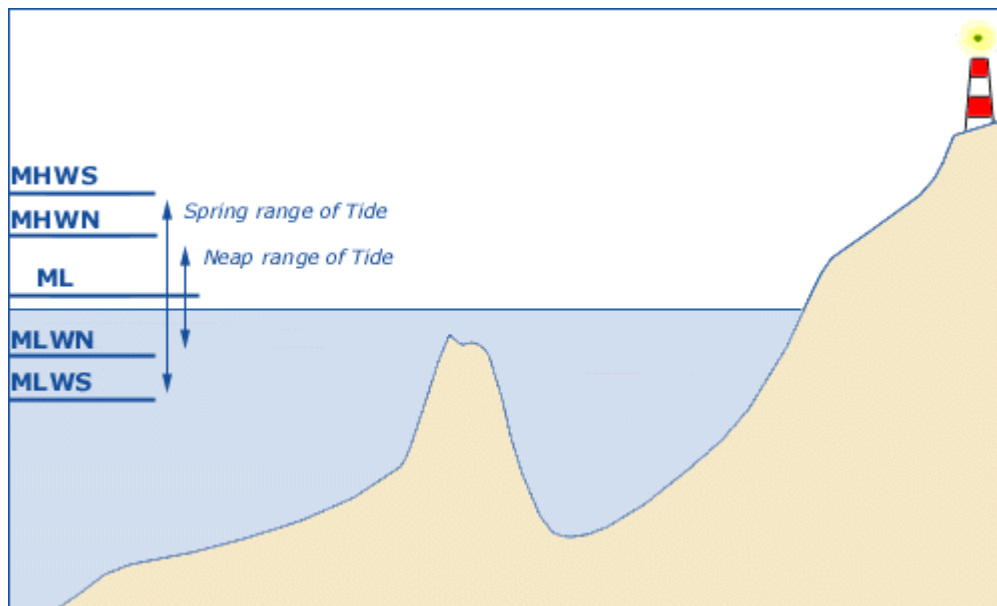
However, a closer look at the orbit of the moon reveals that the moon is not always in the equatorial plane, resulting in three types of tides:

- **Semi-diurnal tide**: Featuring two highs and two lows each day, with minimal variation in the height of successive high or low waters. This type is more likely to occur when the moon is over the equator.
- **Diurnal tide**: Only a single high and a single low during each tidal day; successive high and low waters do not vary by a great deal. This tends to occur in certain areas when the moon is at its furthest from the equator.
- **Mixed tide**: Characterized by wide variations in heights of successive high and low waters, and by longer tidal cycles than those of the semi-diurnal cycle. These tides also tend to occur as the moon moves furthest north or south of the equator.



## Chart Datums

The depths and heights in the chart need a plane of reference: the Chart Datum (see interactive figure below). Depths are usually described with respect to low water reference planes (yielding lower charted depths, which are safer) and heights are shown with respect to high water reference planes (again, yielding lower vertical clearances on the chart, which are safer). As such, the chance that the **observed depth** or vertical clearance beneath a bridge is smaller than the **charted depth** or **height** is rather small.



In this example the Charted Depths are related to LAT. The Observed Depth or Drying Height is a combination of Tidal Height & Charted Depth.

This example shows the various spring and neap tides around mean water level. Note that a spring low water is the lowest. Both ranges are indicated.

In this example the light elevation is reduced to high water. Also a clearance under a bridge is charted in that way. The 'height' refers to the building itself. On land yet another CD can be in use.

Some Chart Datums and their abbreviations:

**MHWS** : Mean High Water Spring

**HW** : High Water

**MHWN** : Mean High Water Neap

**ML** : Mean Level

**MLWN** : Mean Low Water Neap

**MLWS** : Mean Low Water Spring

**LAT** : Low Astronomical Tide

## Overview

**Tide:** The vertical rise and fall of the surface of a body of water caused primarily by the differences in gravitational attraction of the moon, and to a lesser extent the sun, upon different parts of the earth when the positions of the moon and sun change with respect to the earth.

**Spring Tide:** The tidal effect of the sun and the moon acting in concert twice a month, when the sun, earth and moon are all in a straight line (full moon or new moon). The range of tide is larger than average.

**Neap Tide:** This opposite effect occurs when the moon is at right angles to the earth-sun line (first or last quarter). The range of tide is smaller than average.

**Range:** The vertical difference between the high and low tide water levels during one tidal cycle.

**Tidal Day:** 24 hours and 50 minutes. The moon orbits the earth every month, and the earth rotates (in the same direction as the moon's orbit) on its axis once every 24 hours.

**Tidal Cycle:** One high tide plus a successive low tide.

**Semi-diurnal Tide:** The most common tidal pattern, featuring two highs and two lows each day, with minimal variation in the height of successive high or low waters.

**Diurnal Tide:** Only a single high and a single low during each tidal day; successive high and low waters do not vary by a great deal. Such tides occur, for example, in the Gulf of Mexico, Java Sea and in the Tonkin Gulf.

**Mixed Tide:** Characterized by wide variation in heights of successive high and low waters, and by longer tide cycles than those of the semidiurnal cycle. Such tides occur, for example, in the U.S. Pacific coast and many Pacific islands.

**Chart Datum** or **Tidal reference planes:** These fictitious planes are used as the sounding datum for the tidal heights.

**Drying Height:** Clearance in meters (or feet in old charts) above the chart datum.

**Charted Depth:** Clearance in meters (or feet in old charts) below the chart datum.

**Observed Depth:** Height of tide + charted depth: the actual depth in meters.

**Height of light:** The height of light above the bottom of its structure.

**Elevation:** The height of the light above the chart datum.

**Rule of Twelve:** Assuming a tidal curve to be a perfect sinusoid with a period of 12 hours. The height changes over the full range in the six hours between HW and LW with the following fractions during each respective hour: 1/12 2/12 3/12 3/12 2/12 1/12.

**Rule of Seven:** The change from spring range to neap range can be assumed linear, each day the range changes with 1/7th of difference between the spring and neap ranges. Hence, the daily change in range = (spring range - neap range)/7.

# Tides & tidal prediction

## 1 - Information from the chart

Most often the chart presents succinct tide tables for certain positions. These positions are marked with the 'square'. The table below shows us an example for two different positions. The first refers to Cowes (UK), the second to a position south of Cowes.

Position	Heights above LAT			
	Mean HW		Mean LW	
	Spring	Neap	Spring	Neap
Cowes	1.7 m	1.5 m	0.2 m	0.4 m
	5.2 m	4.3 m	0.4 m	1.2 m

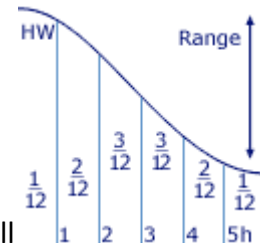
This data only provides us with average high and low waters heights. Moreover, it is merely valid at spring or neap tides. To use it we need to first find out how many hours we are from high water. Secondly, we need to know if it is spring or neap or sometime in between at that particular moment.

We shall use this table to solve two types of problems. Finding height of tide at a particular location at a particular time:

- To get over a shoal.
- To pass under a bridge.

Almanacs and many other nautical publications contain predictions of the times of high and low tides at many major standard ports [\[i\]](#). Also listed are differences in times of tides from these ports for additional secondary ports [\[i\]](#). To work with this succinct data we need two extra tools:

- To interpolate between high and low water heights we use the Rule of Twelve. We assume the tidal curve to be a perfect sinusoid with a period of 12 hours. The height changes over the full range in the six hours between HW and LW.
  - During first hour after HW the water drops  $\frac{1}{12}^{\text{th}}$  of the full range.
  - During the second hour an additional  $\frac{2}{12}^{\text{th}}$ .
  - During the third hour an additional  $\frac{3}{12}^{\text{th}}$ .
  - During the fourth hour an additional  $\frac{3}{12}^{\text{th}}$ .
  - During the fifth hour an additional  $\frac{2}{12}^{\text{th}}$ .
  - During the sixth hour an additional  $\frac{1}{12}^{\text{th}}$ .



Hence, two hours after the HW the water has fallen  $\frac{3}{12}$  of the full range.

- To interpolate between spring and neap tides we use the **Rule of Seven**. Since the change from spring range to neap range can be assumed linear (instead of sinusoid), each day the range changes with  $\frac{1}{7}^{\text{th}}$  of difference between the spring and neap ranges. Hence, the daily change in range is  $(\text{spring range} - \text{neap range})/7$ .

### Shoal problem:

Our shoal near Cowes has a charted depth of 1 meter and we would like to cross it at about 15:00 hours with our yacht (draft 1,5 m).

From any nautical almanac we find that HW occurs at **03:18 15:53** and LW occurs at **09:45 22:03** at a standard port nearby. We also find that at our location HW occurs one hour later and that spring tide is due in two days. Hence, we have a HW around 17:00.

- Via the rule of seven we find out that today the **range** is:

spring range -  $2 \times ((\text{spring range} - \text{neap range})/7)$

$\Leftrightarrow 4,8 - 2 \times ((4,8 - 3,1)/7) \Leftrightarrow 4,8 - 2 \times 0,25 = 4,3 \text{ m.}$

- We also need today's HW height:

which is Spring HW - 2 days x  $(5,2 - 4,3)/7 = 5,0 \text{ m}$  [i](#).

- Via the rule of twelve we find out that at two hours before high water [i](#) the height is:  $5,0 - 3/12 \times 4,3 = \text{height at 15:00 hours} = 3,9 \text{ m}$ .

So, after three interpolations we derive the water height at 1500 hours. Considering the charted depth leads to an observed depth of 4,9 meters, enough for our draft of 1,5 meters.

### Bridge problem:

An overhanging rock, power lines or bridges have their clearances charted with respect to another chart datum than LAT. Normally, 'high water' or 'MHW spring' are used as reference planes.

An example:

Above our shoal hangs the 'Cowes bridge'. At 15:00 hours we would like to pass this bridge, which has a charted height of 20 meters to HW. Our mast is 23 meters high. In the example above we found that the water height was 1,1 meters below HW level at that time.

Obviously, we will have to wait!

So, at what time will we be able to pass under this bridge?

The water height must be 3 meters lower than HW level (5,0 m). That is almost 9/12 of the range (4,3 m) indicating four hours after HW [i](#). Conclusion, we will have to wait at least six hours in total.

## 2 - Information from tide tables

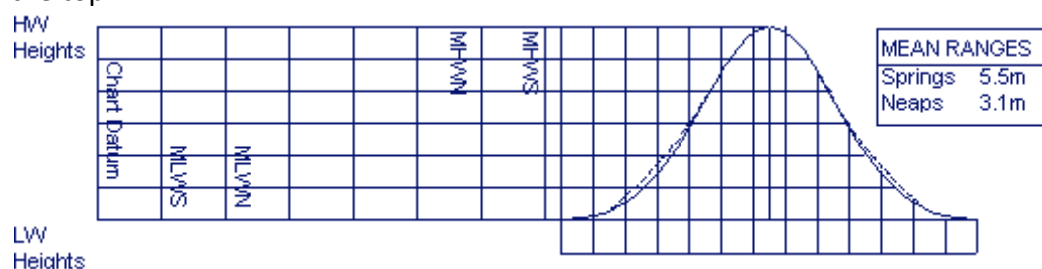
Rotterdam April 2003		Heights in tenth of meters above LAT																									
		Hours																									
Day		00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23		
12		33	28	22	16	11	08	09	13	19	25	32	36	37	33	28	21	15	10	10	12	16	21	27	33		
13		37	35	30	23	17	11	07	08	11	16	23	32	38	39	35	29	22	14	09	08	10	14	20	27		
14		35	39	37	31	24	16	09	06	07	10	16	24	34	41	41	36	29	20	12	07	07	10	14	20		
15		30	39	41	37	31	23	14	07	04	07	11	17	27	39	44	41	35	27	17	09	05	07	11	15		
16		23	35	43	41	36	29	19	10	05	04	08	13	20	33	44	44	39	32	23	13	06	05	08	12		
17		18	28	41	45	40	34	25	15	07	03	05	10	16	25	40	47	43	37	28	18	09	04	05	10		
18		15	21	35	46	44	38	30	20	11	04	03	07	12	19	31	45	47	41	33	23	13	05	03	07		

Instead of mere averages, a tide table provides us each day with the times of high and low water for a particular place. Basically, it is same table like the one we found in the chart, but is extended for every day in a year. By using this method we get more accurate water heights since it involves less interpolation.

The example shows us a part of a very detailed tide table, which even includes heights for every hour.

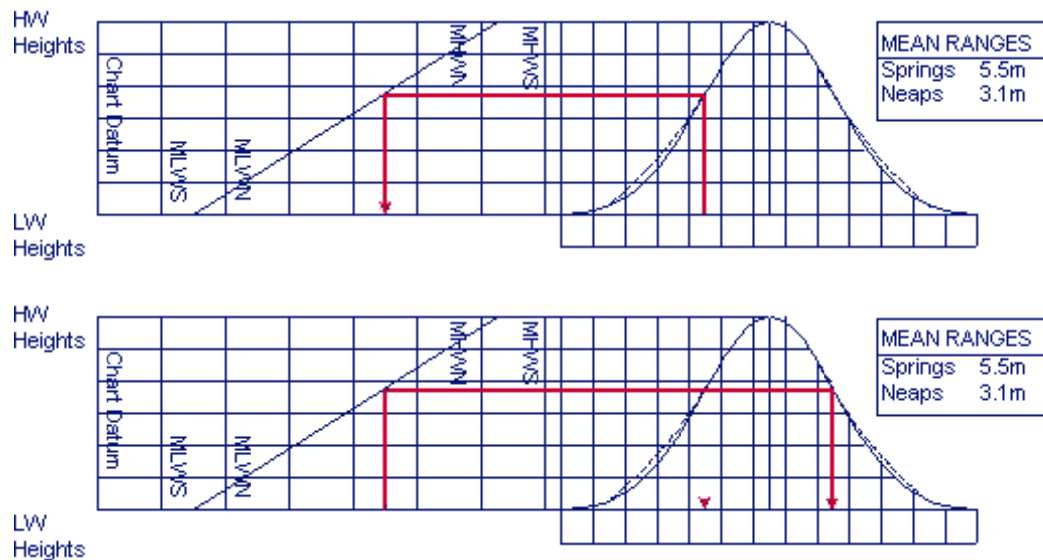
## 3 - Information from tidal curves

In most tables the tides can also be characterized by a tidal curve. This method substitutes the rule of twelve providing more accurate heights. The left side contains the water height information with the lowest heights to the left where also the chart datum is indicated. The low water height will be marked at the bottom and the high water height will be marked at the top.



The area under the curve will be marked with the time information.

To find the water height at a specific time we need to know first how many hours before or after the HW this is.



Often this is done when the curve is not sinusoid and the rule of twelve is rendered useless.

## Overview

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**Rule of Seven:** The change from spring range to neap range can be assumed linear, each day the range changes with  $\frac{1}{7}$ th of difference between the spring and neap ranges. Hence, the daily change in range =  $(\text{spring range} - \text{neap range})/7$ .

# Currents & navigation

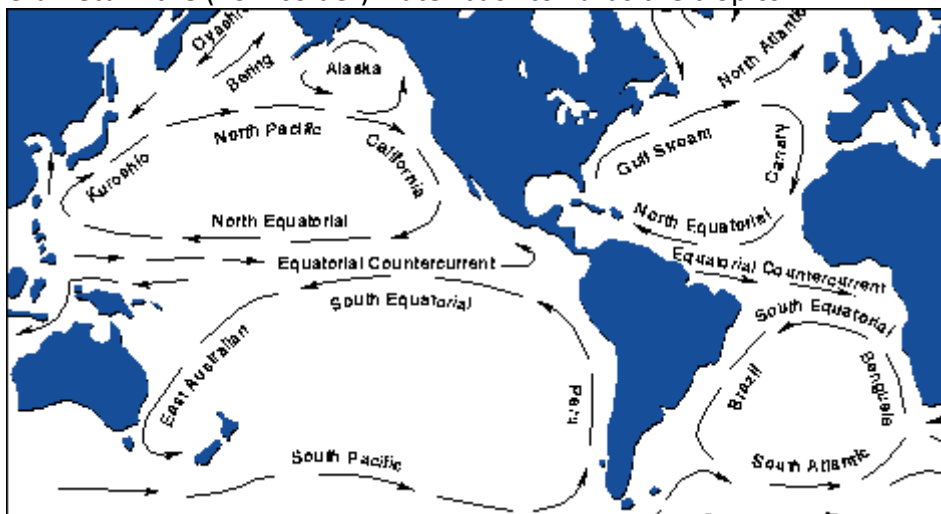
## Currents

Currents reflect the horizontal movement of water whereas tides reflect vertical movements. These currents influence the ship's position and are therefore important to understand.

The horizontal movement is primarily caused by the gravitational pull of celestial bodies. But also other factors are in play:

- differences in water temperatures caused by heating and cooling due to the earth's atmosphere;
- differences in salinity caused by rain, evaporation and estuaries;
- wind induced friction;
- the Coriolis force which is a consequence of the earth's rotation.

Prominent features in the map of the major **oceanic surface currents** include the subtropical **gyres** centered on 30 degrees latitude in each of the major ocean basins. The earth's rotation (origin of the Coriolis force) and the change in wind direction with latitude (from the east in the tropics and from the west at mid-latitudes) cause the circulation of the gyres to be clockwise in the Northern Hemisphere and counterclockwise in the Southern Hemisphere. The well-known Gulf Stream in the Atlantic and its counterpart in the Pacific, the Kuroshio Current, are strong currents [i](#) that carry heat northward from the tropics. The **deep oceanic currents** (not shown) are caused primarily by water density differences and in general return the (now colder) water back towards the tropics.



Click chart to see the whole world!

To predict the behavior of major ocean currents several references are available. The **Sailing Directions Planning Guides** contain some information on normal locations and strengths of ocean currents. Nevertheless, the **Pilot Charts** are by far the best reference for predicting the direction and speed of these currents. On these charts, arrows indicate the direction of the prevailing current; a number printed above the arrow indicates the average speed. Since this information is based upon historical averages, it won't predict the actual ocean current encountered with 100% accuracy.

Ocean surface currents need not be considered in coastal areas. Usually, when close to the continental shelf, the horizontal movement of water is defined by two terms:

**tidal stream** or **tidal current**: gravitational

**current:** gravitational, rivers, wind

In order to predict tidal stream one needs to use tide tables in conjunction with a **tidal atlas**, or a **chart diamond**.

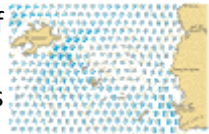
Tidal streams are described by **drift/rate** and **set**, in which drift/rate is the speed and set is the direction of the current.

## Tidal Atlases

Tidal stream atlases show the tidal currents for each hour of the tidal cycle. They comprise a total of 13 tidal charts ranging from 6 hr before HW till 6 hr after HW [\[1\]](#). So, these charts are relative to the time of HW and to use them we must know the absolute time of HW.

Though several layouts can be used, usually the direction of the tidal stream is shown by arrows, which are heavier where the tidal streams are stronger.

Figures against the arrows give the mean neap and spring drift or **rate** in tenths of knots. For example,  $\frac{21.46}{\rightarrow}$  indicates a mean neap drift of 2.1 knots and a mean spring drift of 4.6 knots.



# Lighthouses and buoys

## Navigation aids

**Navigation aids** are special structures like lighthouses, lightships, beacons, buoys, etc that are used to enhance safety by providing more opportunities to obtain LOPs.







These lights and marks are prescribed across the world by the International Association of Lighthouse Authorities (IALA). In 1977 this **IALA** endorsed two maritime buoyage systems putting an end to the 30 odd systems existing at that time. Region A (**IALA A**) covers all of Europe and most of the rest of the world whereas region B (**IALA B**) covers only the America's, Japan, the Philippines and Korea. Fortunately, the differences between these two systems are few. The most striking difference is the **direction of buoyage** (see Lateral buoys below). All marks within the IALA system are distinguished by:







- Shape
- Colour
- Top mark
- Light

## Light Identification

During daytime identification of navigation aids is accomplished by observing: **location, shape, colour scheme, auxiliary features** (sound signals, RACON [Li](#), RC [Li](#), etc) or **markings** (name, number, etc).

During the night, we use the features of the navigation aid's **light** to both identify it and ascertain its purpose. There are three features to describe the light:

- **Colour**: Either white, red, green or yellow. If no colour is stated in the chart, default is white.
- **Period**: The time in seconds needed for one complete cycle of changes.  
 The arrow indicates the 10 second period of this flashing light 'Fl(3) 10 s'.
- **Phase characteristic**: The particular pattern of changes within one complete cycle (hence, within one period). Below are the most common types:
  - **Fixed (F)**  
 This light shines with an unblinking and steady intensity and is always on. In this example a yellow fixed light is shown.
  - **Flashing (Fl)**:  
 The duration of the light is always less than the duration of the darkness. The frequency does not exceed 30 times per minute.
  - **Quick Flashing (Q)**:  
 Again, the duration of quick flash is less than the darkness. The frequency is at least 60 times per minute.
  - **Very Quick Flashing (VQ)**:  
 Also here, the duration of very quick flash is less than the darkness. The frequency is at least 100 times per minute.
  - **Interrupted Quick Flashing (IQ)**:  
 Like Quick Flashing with one moment of darkness in one period.

- **Isophase (Iso):**
-  This light has equal duration between light and darkness. A period consists of both a light and a dark interval. Also called Equal Interval (E Int).
- **Group Flashing (Gp Fl(x+x)):**
-  This is actually a combination of two patterns in one period. In this example the first 2 flashes followed by the pattern of 3 flashes result in 'Gp Fl(2+3)'.
- **Occulting (Occ):**
-  Occulting is the opposite of flashing, the light is more on than off.
- **Alternating (AL):**
-  An alternating light changes colour. This special purpose light is typically used for special applications requiring the exercise of great caution. In this example ALT.WG is shown, alternating between green and white.
- **Morse "U" (Mo (U)):**
-  This light shows two flashes and a longflash, which is equivalent to the letter "U" in Morse code.
- **Long-Flashing (LFL):**
-  This light has one long flash in a period. A long flash is at least 2 seconds long.

Let's look at some examples using colour, period and phase characteristics. The arrows mark the periods:

-  **Fl (4) 8 s**
-  **Oc (2+3) 10 s**
-  **Iso G 4 s**

All lighted navigation aids are either **major** or **minor lights**, where major lights are used for key navigational points along seacoasts, channels and harbour and river entrances. These lights are normally placed in lightships, lighthouses and other permanently installed structures, providing both high intensity and high reliability of the lights. Major lights are then subdivided in **primary lights** (very strong, long range lights used for the purpose of making landfalls or coastal passages) and **secondary lights** (shorter range lights found for example at harbour and river entrances). Important details of (especially) primary lights can be found in a reference called **the Light List** where information (about pedestals etc.) can be found which is not included in the chart.

Minor lights on the other hand are likely to be found within harbours, along channels and rivers. These have a low to moderate intensity and sometimes mark isolated dangers.

## Five types of navigation buoys:

- Lateral
- Cardinal
- Isolated danger
- Safe water
- special

## Lateral Buoys and Marks

The location of lateral buoys defines the borders of channels and indicates the direction. Under IALA A red buoys mark the port side of the channel when returning from sea, whereas

under IALA B green buoys mark the port side of the channel when sailing towards land. Red buoys have even numeration plus red lights and green buoys have odd numeration plus green lights. Lateral lights can have any calm phase characteristic except FL (2+1).



Generally, when two channels meet one will be designated the preferred channel (i.e. most important channel). The buoy depicted on the right indicates the preferred channel to starboard [\[i\]](#) under IALA A. The light phase characteristic is R FL (2+1):

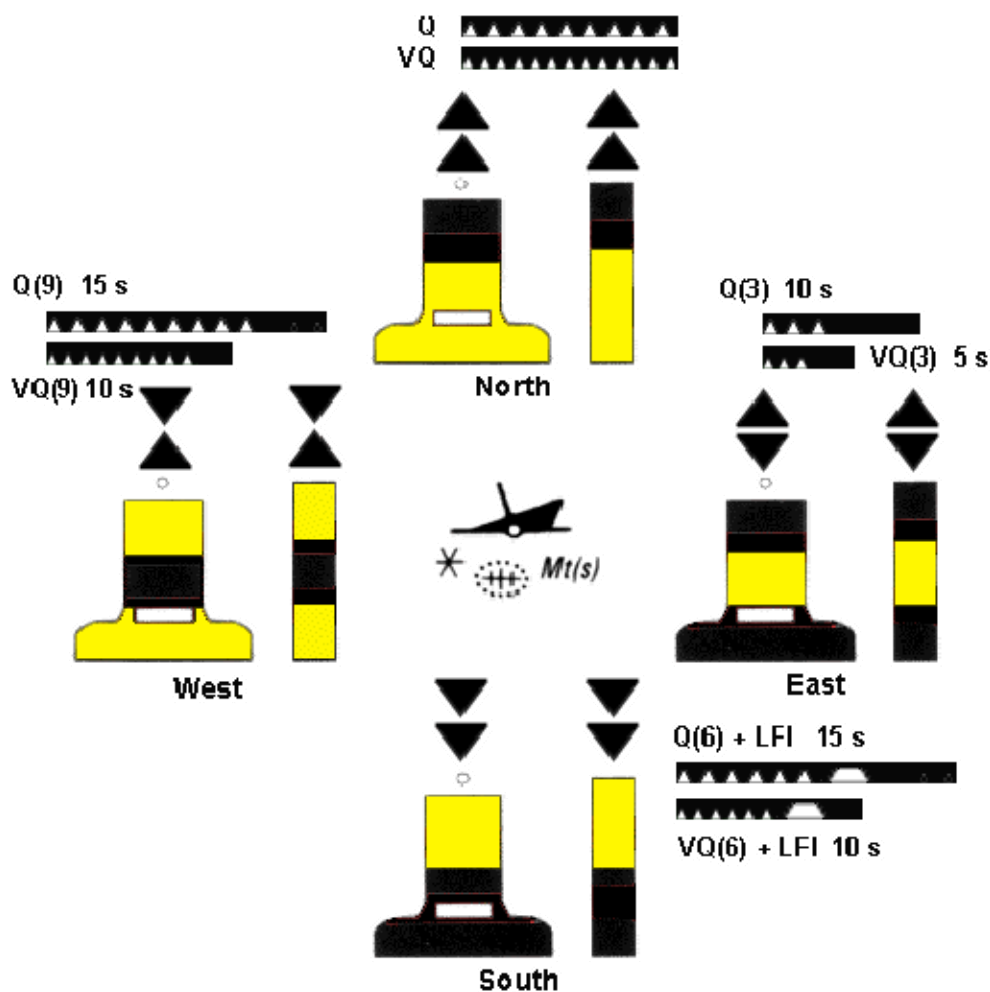


The buoy depicted on the left indicates the preferred channel to port [\[i\]](#) under IALA A. These buoys are marked with the numerations of both channels. The light phase characteristic is G FL (2+1):

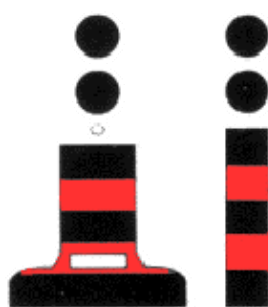


## Cardinal Buoys

The four cardinal buoys indicate the safe side of a danger with an approximate bearing. For example, the West cardinal buoy has safe water on its West and the danger on its East side. Notice the 'clockwise' resemblance of the light phase characteristics. The topmarks consist of two black triangles placed in accordance with the black/yellow scheme of the buoy. When a new obstacle (not yet shown on charts) needs to be marked, **two** cardinal buoys will be used to indicate this 'uncharted' danger. The cardinal system is identical in both the IALA A and IALA B buoyage systems.



## Marks indicating Isolated dangers

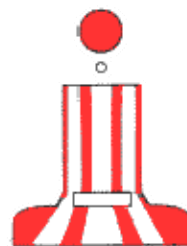


This type of buoy indicates the position of an isolated danger, contrary to cardinal buoys which indicate a direction away from the danger. The light (when present) consists of a white group flash: Fl(2).

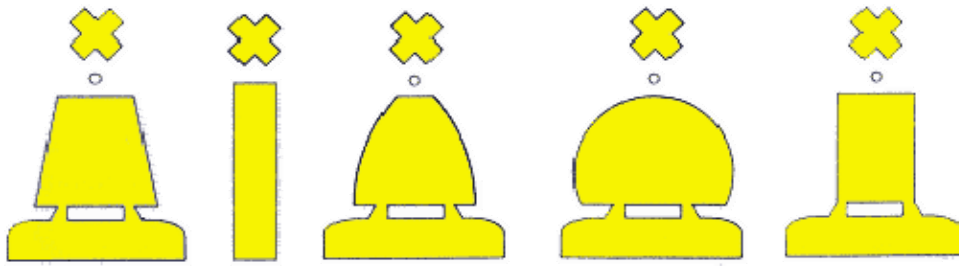


## Marks indicating Safe water

Notice that whereas most horizontal striping 'spells danger', this safe water buoy is vertically striped. These marks are for example seaward of all other buoys (lateral and cardinal) and can be used to make landfall. Lights are usually calm and white.



## Special Buoys and Marks



We saved these buoys for last since they have not an actual navigation purpose. Most of the time these yellow buoys indicate areas used by navies or pipelines or surfing.

## Range

It is important to know at what distance (range) we may see a certain light, and when we can expect to lose sight of it, especially when making landfall. This **range** can be defined in several ways:

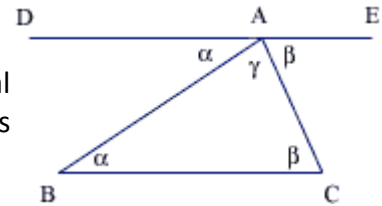
**Charted or Nominal Range:** The nominal range is indicated in the chart next to the light or can be found in the Light List. This is the maximum distance at which a light may be seen at night based upon intensity and 10 nautical miles of visibility.

Use the logo to navigate through this course.

# Mathematics: Running fixes

## The sum of angles in a triangle is $180^\circ$

Let line **DAE** be parallel to line **BC**, then the angles  $\alpha$  and  $\alpha$  equal angles **DAB** and **EAC**, respectively. Therefore, the sum of angles in the triangle is  $180^\circ$  : a straight line.



## “Doubling the angle” yields two equal angles

So,  $\alpha + \delta + \gamma = 180^\circ$

$\alpha + 180 - \beta + \gamma = 180^\circ$

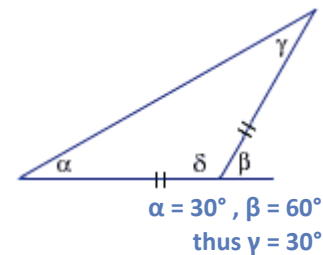
$2\alpha = \beta$

$\alpha + 180 - 2\alpha + \gamma = 180^\circ$

$180^\circ - \alpha + \gamma = 180^\circ$

$-\alpha + \gamma = 0$

$\gamma = \alpha$



## Two equal angles render an triangle isosceles

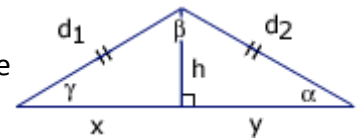
In the triangle on the right,  $\alpha = \gamma$  and  $\beta = 2\alpha$ .

By constructing the bisector **h** of angle  $\beta$  we create two little triangles

in which  $x=y$ .

Therefore,  $d_1=d_2$ .

Next math chapter: **Distance of horizon**



# Mathematics: Distance of horizon

## Distance of horizon

**AD = h** is the height of eye above the earth.

**DO = BO = CO = r** (radius of the earth).

*Factum:* **any angle between a tangent line to a circle and the radius of the circle is a right angle.**

Since we have a right triangle **ABO** where **AB = d**,

**AO = h+r** and **BO = r**,

we can find a formula for **d** in terms of **h**:

$$(AO)^2 = AB^2 + BO^2$$

$$(h+r)^2 = d^2 + r^2$$

$$d = \sqrt{(h+r)^2 - r^2},$$

where **r** is approx. 3.440.1 nm

An example: Let the eye height (**h**) be 4 meters (=0.0022nm); find the distance in nm of the **geometrical horizon**.

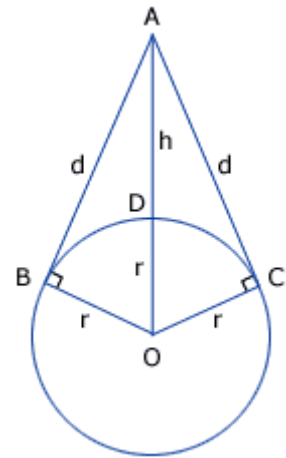
$$d = \sqrt{(0.0022 + 3.440.1)^2 - 3.440.1^2} ; d = \sqrt{11834303 - 11834288}$$

$$d = \sqrt{15.146} ; d = 3.89 \text{ nm (geometrical)}$$

The distance of the **visible horizon** as found in the table is greater (4.2 nm) due to atmospheric refraction.

The semi-empirical function used is:

$$d = \sqrt{(2 \times 3440.1 \times h) / (1852 \times \rho_o)}, \text{ where } \rho_o \text{ accounts for refraction (0.8279).}$$



# Mathematics: Sextant angles

## Vertical sextant angle

The triangle OBL (see fig. below) can be described in terms of H,  $\alpha$  and Distance:

$$\text{Distance} = H / \tan(\alpha)$$

The angle in rad. ( $0-2\pi$ ) and both height and distance in metres.

From rad. to degrees:  $\alpha = A * \pi / 180$ , 'A' being the same angle in degrees.

To describe angle A in minutes total, then  $A * 60 = a$ , thus  $\alpha = (a/60) * (\pi/180)$ . So,  $\alpha = a/3438$ , 'a' being the angle in arc minutes.

*FACTUM*:  $\tan(x) = x$ , if angle x is small.

Resulting in (with  $\pi = 3.14$ ): **Distance (m) =  $H * 3438/a$**

Furthermore, distance in nm. = distance in meters/1852.

Voilà, la very practical equation:

**Distance =  $1.856 * H/a$**

It contains just two approximations, both of negligible influence. First, we left out the *tan* function and second we used 3.14 for  $\pi$ .

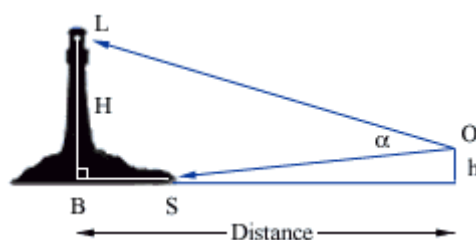
Please realize that a smaller angle improves the

approximation of the *tan*. Yet, as an opposing effect the instrument error of a smaller sextant angle increases.

All in all, the factor 1.856 is *not* a typo, and just by chance near to the nautical mile: 1.852 kilometres. If you are still reading, you are very brave person and might perhaps agree that it originates from:  $(60 * 180) / (\pi * 1852)$ .

So far we considered a perfect triangle (OBL) and forgot that life isn't always perfect. Height h is usually quite small, but distance SB sometimes is not. This leads to an extra premise, which is seldom mentioned by other navigation textbooks:

**Angle OLS should be bigger than  $15^\circ$ .**



In the beginning of the 19th century, sailors were describing wind forces instead of wind speeds, yet they lack a uniform scale to estimate these forces.

His scale as well as the more precise wind speeds in Knots or sometimes in meters per second are now widely used.

Beaufort scale									
Specification and speeds			Equivalent speed at 10 m above sea level				Description in forecasts	State of sea	Probable height of waves* metres
Force	Description	Specification for use at sea*	Mean <i>knots</i>	<i>metres per second</i>	Limits <i>knots</i>	<i>metres per second</i>			
0	Calm	Sea like a mirror.	0	0.0	<1	0.0-0.2	Calm	Calm	0.0
1	Light air	Ripples with the appearance of scales are formed, but without foam crests.	2	0.8	1-3	0.3-1.5	Light	Calm	0.1 (0.1)
2	Light breeze	Small wavelets, still short but more pronounced. Crests have a glassy appearance and do not break.	5	2.4	4-6	1.6-3.3	Light	Smooth	0.2 (0.3)
3	Gentle breeze	Large wavelets. Crests begin to break. Foam of glassy appearance. Perhaps scattered white horses.	9	4.3	7-10	3.4-5.4	Light	Smooth	0.6 (1.0)
4	Moderate breeze	Large waves begin to form; the white foam crests are more extensive everywhere. Probably some spray.	13	6.7	11-16	5.5-7.9	Moderate	Slight	1.0 (1.5)
5	Fresh breeze	Moderate waves, taking a more pronounced long form; many white horses are formed. Chance of some spray	19	9.3	17-21	8.0-10.7	Fresh	Moderate	2.0 (2.5)
6	Strong breeze	Large waves begin to form; the white foam crests are more extensive everywhere. Probably some spray.	24	12.3	22-27	10.8-13.8	Strong	Rough	3.0 (4.0)
7	Near gale	Sea heaps up and white foam from breaking waves begins to be blown in streaks along the direction of the wind.	30	15.5	28-33	13.9-17.1	Strong	Very rough	4.0 (5.5)
8	Gale	Moderately high waves of greater length; edges of crests begin to break into spindrift. The foam is blown in well-marked streaks along the direction of the wind.	37	18.9	34-40	17.2-20.7	Gale	High	5.5 (7.5)
9	Strong gale	High waves. Dense streaks of foam along the direction of the wind. Crests of waves begin to topple, tumble and roll over. Spray may affect visibility.	44	22.6	41-47	20.8-24.4	Severe gale	Very high	7.0 (10.0)
10	Storm	Very high waves with long overhanging crests. The resulting foam, in great patches, is blown in dense white streaks along the direction of the wind. On the whole, the surface of the sea takes a white appearance.The 'tumbling' of the sea becomes heavy and shock-like. Visibility affected.	52	26.4	48-55	24.5-28.4	Storm	Very high	9.0 (12.5)

<b>11</b>	<b>Violent storm</b>	Exceptionally high waves (small and medium-sized ships might be lost to view for a time behind the waves). The sea is completely covered with long white patches of foam lying along the direction of the wind. Everywhere the edges of the wave crests are blown into froth. Visibility affected.	60	30.5	56-63	28.5-32.6	Violent storm	Phenomenal	11.5 (16.0)
<b>12</b>	<b>Hurricane</b>	The air is filled with foam and spray. Sea completely white with driving spray; visibility seriously affected.	-	-	64 and over	32.7 and over	Hurricane force	Phenomenal	14.0 (-)

\* These columns are a guide to show roughly what may be expected in the open sea, remote from land. Figures in brackets indicate the probable maximum height of waves. In enclosed waters, or when near land with an offshore wind, wave heights will be smaller and the waves steeper.